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Using Magic Board as a Teaching Aid in Third Grader Learning of Area Concepts

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ABSTRACT

The purpose of this study was to explore the impact of incorporating Magic Board in the instruction of concepts related to area. We adopted a non-equivalent quasi-experimental design and recruited participants from two classes of third-grade students in an elementary school in Taoyuan County, Taiwan. Magic Board was used as a teaching aid in the experimental group, and physical manipulatives were employed as teaching aids in the control group. Both groups took the Basic Area Concept Test as a pretest, followed by the Area Concept Test to evaluate retention two weeks later. Results demonstrate the effectiveness of Magic Board over that of physical manipulatives on three subscales of immediate learning performance and all four subscales of retention performance. We also discuss the implications of these results and provide recommendations for future research.

Keywords

Virtual manipulatives, Mathematics teaching, Elementary school students

Introduction

The use of Physical manipulatives

Physical manipulatives are physical objects, such as base-ten blocks, algebra tiles, Unifix Cubes, Cuisenaire rods, fraction pieces, pattern blocks, and geometric solids, which are commonly used in mathematics education to make abstract ideas and symbols more meaningful and comprehensible to students (Durmus & Karakirik, 2006). Clements (2000) proposed physical manipulatives to help students construct, develop, and integrate a variety of concepts and their mathematical representations. Many studies have found that students who use physical manipulatives to explore mathematical concepts outperform those who do not (Fennema, 1972; Zacharia & Olympiou, in press). Through meta-analysis, Suydam and Higgins (1976) verified that lessons involving physical manipulatives can lead to more significant achievements than lessons without. They suggested the following guidelines for the use of physical manipulatives: (1) frequent use in a comprehensive mathematics program, consistent with program goals; (2) use in conjunction with other aids (such as pictures, diagrams, textbooks, films, and similar materials); (3) use in a manner appropriate to the nature of the mathematical content; (4) use in conjunction with exploratory and inductive approaches; and (5) use with programs that encourage symbolical recording of the results. Moyer (2001) identified a number of reasons for the infrequent use of physical manipulatives to teach mathematics in the classroom. First, teachers are typically required to purchase physical manipulatives, which can be costly, or to make them themselves, which takes considerable time. Second, using physical manipulatives in a real classroom poses a number of difficulties, including classroom control, cleaning, and storage (Yuan, Lee, & Wang, 2010). For teachers who would otherwise not use physical manipulatives, information technology enables the creation of virtual learning environments in which to address these problems.

The use of virtual manipulatives

Moyer, Bolyard, and Spikell (2002) recently defined a virtual manipulative as an interactive, web-based visual representation of a dynamic object that facilitates the development of mathematical concepts in students. Virtual manipulatives are generally more than just electronic replications of their physical counterparts. Clements and

McMillen (1996) considered virtual manipulatives as useful tools for learning when they possess the following features: (1) allow students to change, repeat, and undo actions in a straightforward manner; (2) allow students to save configuration and action sequences; (3) dynamically link various representations and maintain a tight connection between pictured objects and symbols; (4) allow students and teachers to pose and solve problems; and (5) enable students to control flexible, extensible tools for the development of mathematical concepts. Such virtual manipulatives serve many purposes and help in the formation of connections between mathematical ideas (Lee & Chen, 2009). However, studies have identified a number of limitations and disadvantages of this approach (Highfield & Mulligan, 2007; Hunt, Nipper, & Nash, 2011). First, the computer skills required to use virtual manipulatives have proven challenging for students. Without teacher support, scaffolding, and practice, students are unable to manipulate virtual objects on screen. Second, virtual manipulatives can distract some students from the problem at hand. Finally, students are unable to actually touch virtual manipulatives, which deprives students of the tactile experience available to students using physical manipulatives (Olkun, 2003).

Introduction of magic board

Magic Board, developed by Yuan, Chen, and Chang (2007), is a well-known web-based virtual environment for teaching elementary mathematics, based on experience obtained in the development of virtual manipulatives (it can be accessed at http://163.21.193.5). The English version of Magic Board is available from the upper-right corner of the website home page by clicking the “Try English version” button. Magic Board comprises three important components: Magic Board Software, Problem Posing Center, and Instructional Material Center. Users that register as members of Magic Board are eligible to use all of the components to construct and share instructional materials on the Magic Board platform. Users can access shared materials and adapt them to their own classes. Non-registered users (guests) can open and use Magic Board Software to adopt existing instructional materials for implementation into their instruction. All components and graphic files in Magic Board are directly downloadable for use with related worksheets.

Magic Board Software (MBS)

Magic Board Software provides a tool box that includes a wide range of frequently used elementary mathematics manipulatives that instructors may access instantly while teaching (Fig. 1). Clicking the tool box button in the function button area hides or shows the tool box in the display area. Teachers can drag these objects to the display area and a right-click allows the user to control the properties. In the function button area, word addition is used to enter text to present word problems, the doodle pen to scribble or mark up anything on the screen, and the clean doodles button for erasing. The cheering button is used to encourage students who are doing well or to motivate students who are struggling. The background change enables a rapid change of backgrounds. Clicking the rubbish bin clears all components from the display area.

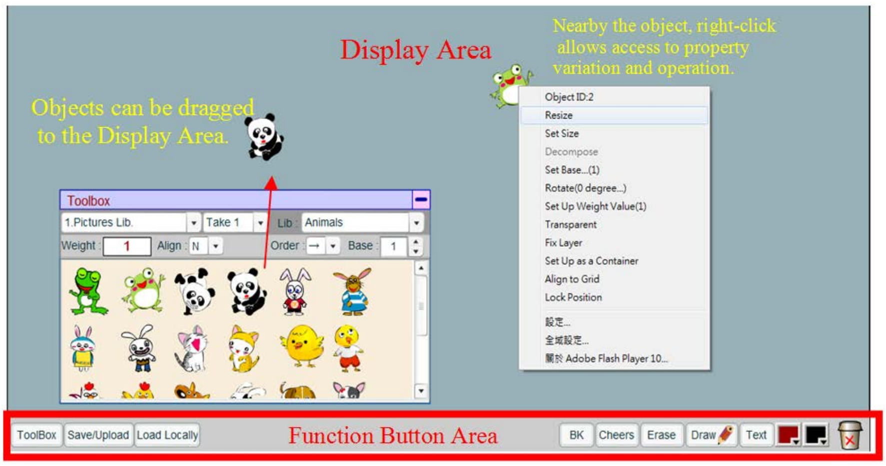

Figure 1. Magic Board software interface

Problem Posing Center (PPC)

Users can log into the Magic Board platform to access resources from the PPC and search for shared problems according to the level of their students and mathematical content. Users can save and upload teaching materials by clicking on Edit and Upload, and following the instructions to enter the grade of the target students and the content used in their lessons. Users can search for problems based on these pre-set search values. Clicking on My Posed Problems lists past problems uploaded by Magic Board members (Fig. 2).

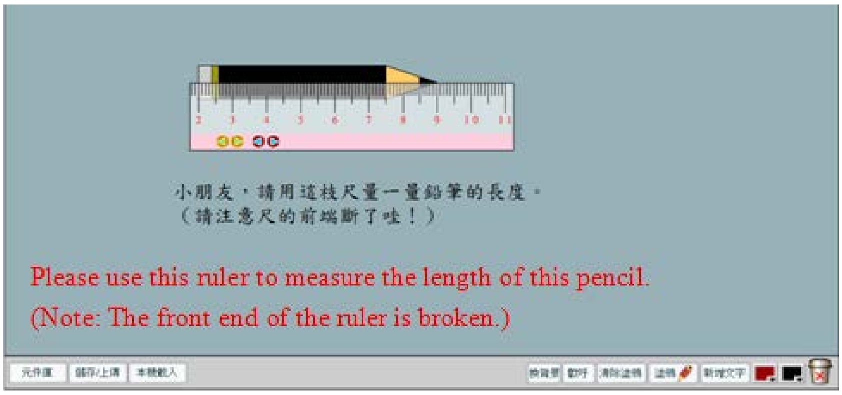

Figure 2. Length measurement problem posed by a user

Instructional Material Center (IMC)

The Search function enables users to search for shared instructional materials under various classifications, such as the grade of the students the material is intended for and the nature of the mathematical concepts being discussed. Clicking on Organize Instructional Materials enables users to browse through problems and select suitable source materials with which to personalize curricula. My Instructional Materials reveals a list of lessons and materials previously uploaded by Magic Board members. After entering Organize Instructional Materials (Fig. 3), users search through problems numbered according to their selection order. Clicking on Set Posing Problems and entering the classification information of the instructional materials completes the compilation. The arrows in the Function Button Area control the presentation of the comprehensive instructional materials available at the IMC.

Figure 3. Operating interface of organized instructional materials

Learning of area concepts in elementary-school mathematics

The measurement of area is an important topic in school mathematics, closely associated with real-world applications in science and technology (Huang & Witz, 2011). However, recent assessments of mathematical achievement indicate that elementary-school children are failing in tests on the measurement of area (Martin & Strutchens, 2000). Traditionally, instructors have taught the formulas required to measure the area of basic shapes, with an emphasis on performing these tasks efficiently. An overemphasis on the calculation of area using formulas prevents students from gaining adequate experience visualizing geometric figures and becoming familiar with the properties on which the formulas are based. The instruction procedures employed in tiling activities do not directly demonstrate multiplication (Van de Walle, 2004). A failure to provide children with guided exploration often prevents students from grasping the significance of array structures directly through tiling operations. When children are unable to see multiplication properties embedded in array structures, they tend to add the units on the sides of the figures without considering the units within the edges (Schifter, Bastable, Russell, & Woleck, 2002). Educators in mathematics (Fuys, Geddes, & Tischler, 1988; Burns & Brade, 2003) suggest that 2D geometry, including knowledge of the properties of basic shapes, congruence, and geometric motions (flips, turns, translations, decomposition, and re-composition) should form the basis of constructing concepts related to the measurement of area. They also advocate strengthening the understanding of 2D geometry to develop children's knowledge of the formulas used for the measurement of area and an understanding of the relationships among them. Manipulatives often aid in strengthening the concepts of congruence and transformation and facilitating an understanding of the rationale underlying the common formulas used to measure area (Yuan, Lee, & Huang, 2007). The current study designed materials integrated with physical or virtual manipulatives for teaching the measurement of area. The teaching materials covered four topics: the preservation of area, comparison of area using a nonstandard unit, indirect comparison of area, and measuring area using multiplication.

Comparisons between virtual and physical manipulatives

Empirical evidence related to the use of virtual manipulatives for the instruction of mathematics and science in the classroom is still new and somewhat limited. Yuan, Lee, and Wang (2010) examined the influence of polyomino exploration by junior high-school students using virtual and physical manipulatives. In that study, the group using virtual manipulatives learned as effectively as the group using physical manipulatives; however, students in the physical group reduced their strategy more than those in the virtual manipulative group. Manches, O'Malley, and Benford (2010) also indicated that differences in manipulative properties between virtual and physical groups might influence the numerical strategies of children. Triona and Klahr (2003) found that students internalize objects related to science equally when taught using either virtual or physical materials, as long as the method of instruction is preserved. Zacharia and Olympiou (in press) investigated whether physical or virtual manipulative experimentation can differentiate physics learning using four experimental conditions: physical manipulative experimentation (PME), virtual manipulative experimentation (VME), and two sequential combinations of PME and VME, as well as a control (i.e., traditional instruction without PME or VME). Results reveal that the four experimental conditions were equally effective in promoting the conceptual understanding of heat and temperature and all four approaches outperformed the conventional (control) approach.

Reimer and Moyer (2005) reported that third-grade students learning fractions with virtual manipulatives showed statistically significant gains in the development of conceptual knowledge. Student surveys and interviews indicate that manipulatives provided immediate and specific feedback, were easier and faster to use than traditional methods, and enhanced student enjoyment while learning. A comparison of the aforementioned studies provided mixed results, and fewer studies have examined retention as a learning outcome resulting from the use of virtual and physical manipulatives. Although virtual manipulatives generated an exciting range of new possibilities to support learning, fewer studies showed how this more indirect form of virtual representations influence children's interaction compared with direct manipulation of physical objects. Understanding the properties of virtual manipulatives and how these relate to learning is not only important in trying to predict which material will be more beneficial but can help inform the design of new learning material. Therefore, this paper explores the effects of using Magic Board as a teaching aid on the conceptual learning of third-grade students in Taiwan. Teacher observations and class journals are used to explain the learning outcomes between virtual and physical manipulatives and to provide more details about how these materials impact on students' understanding of mathematics concepts. The current research focuses on the following two questions:

1. Does the use of Magic Board have a more positive effect than physical manipulatives on the immediate learning performance of students studying concepts related to area?
2. Does the use of Magic Board have a more positive effect than physical manipulatives on the retention performance of students studying concepts related to area?

Methodology

Research design and participants

This study applied the non-equivalent group pretest/posttest quasi-experimental design to compare learning outcomes (immediate learning and retention) between physical and virtual manipulatives in the study of concepts related to area. Participants included 59 third-grade students (32 boys and 27 girls), from two classes in an elementary school in Taoyuan County, Taiwan. Researchers randomly selected one class as the experimental group (32 students in the virtual manipulative group) and the other class as the control group (27 students in the physical manipulative group). Both groups studied identical materials and engaged in the same learning activities. The only difference was that the experiment group used the Magic Board, while the control group used physical manipulatives. All participants took the Basic Area Concept Test as a pretest and the Area Concept Test as a posttest. Two weeks later, both groups were administered the same Area Concept Test to evaluate retention. One-way analysis of covariance (ANCOVA) was conducted to examine the effects on the immediate learning and retention of concepts related to area between the groups using physical and virtual manipulatives, with the pretest as the covariate. The observations of teachers and a class journal based on video recordings of each teaching session were collected to help explain or elaborate on the quantitative results.

Materials

The teaching materials in this study were adapted from the mathematics textbook used by third-grade students. The content of the instructional material was reviewed by five experienced elementary-school teachers with pedagogical and professional knowledge of mathematics teaching. Minor revisions to the design of instructional materials were made according to the suggestions of the teachers. Table 1 shows the course content and course schedule for each week. Magic Board was used as a teaching aid in the experimental group, and concrete manipulatives were used as teaching aids in the control group. Table 2 shows the differences between the virtual and physical tools.

*Table 1.*Course content and study schedule

Week	Activity	Content
Week 1	Introduction to area	Understanding the concept of area
Week 2	Cutting and combining	Preserving area
		Direct comparison of area
Week 3	Comparing area	Indirect comparison of area
		Comparison of area using a nonstandard unit
Week 4	Understanding one square centimeter	Understanding the meaning of one square centimeter
Week 5	Calculating area	Counting the area with a multiplication approach
Week 6	Applying the concept of area	Estimating the area of an object using the unit of one square centimeter

Table 2. Comparison between the physical and virtual manipulative environment

Tools	Virtual manipulative environment	Physical manipulative environment
Area board	Changes can be made to the color and size of the area.	Changes cannot be made to the color or size of the area.
Grid board	Changes can be made to the color and size of the grid.	Changes cannot be made to the color or size of the grid.
Nail board	Changes made to the color and the size of the object can be examined.	Changes made to the color and the size of the object cannot be examined.
Ruler	The shape of the ruler can dilate and	The shape of the ruler cannot be

	shrink.	changed.
Hints and Show buttons	Pressing the buttons at any time reveals the instructional materials.	Prearranging the instructional materials in a suitable manner.

Instruments

Basic Area Concept Test (BACT)

The BACT was employed to evaluate the prior knowledge of students related to basic concepts of area prior to the experimental program. BACT was based on the task analysis of prior experiences of materials for third-graders dealing with concepts of area. The content of BACT did not include the intervention materials for third-graders. This instrument comprised twenty questions that measure the basic concepts of area required for experimental instruction, including four types of questions: identifying and classifying simple 2D geometric figures; cultivating a sense of length; estimating and preserving the concept of area; and comparing the size of various areas. For each type of question, students were presented with five questions and each correctly answered question scored five points. A number of sample items from BACT are shown in Appendix 1. The content validity of BACT was ensured by expert reviews from three elementary school mathematics teachers and two professors in a related domain. A pilot test had been conducted on BACT the previous academic semester with a group of students, including 62 third-graders with academic backgrounds similar to those of the target audience in this study. Minor revisions to test items were made according to the results of the pilot study. The reliability coefficient of BACT was 0.83 (Cronbach's alpha); therefore, BACT was adopted as the pretest of this study.

Area Concept Test (ACT)

The ACT was used to examine the learning outcomes (immediate learning and retention) of students following the instructional program. The ACT was based on materials for third-graders dealing with concepts of area. The test comprised 20 questions based on the following four subscales: (1) Preserving area: students understand that the area of the object remains the same under translation and rotation, as well as after flipping the object or cutting the object and recombining it; (2) Area comparison using a nonstandard unit: students use the area of one object as a unit to compare the area of other objects; (3) Indirect comparison of area: students copy the area of one object and compare it with the area of another object when the areas of two objects cannot be compared directly; (4) Measuring the area using multiplication: students calculate the area of objects using multiplication rather than addition. Each subscale comprised five questions and each question scored five points. A number of sample items of the ACT are shown in Appendix 2. A pilot test had been conducted on the same group of students using the ACT in the previous academic semester. This included 28 third-grade students with academic backgrounds similar to those of the target audience in this study. Minor revisions to test items were made according to the results of the pilot study. The reliability of ACT was 0.85, as measured by Cronbach's alpha; therefore, ACT was used as an immediate posttest and retention posttest in this study.

Results

Research Question 1: Does the use of Magic Board have a stronger positive effect than physical manipulatives on the immediate learning performance of students studying concepts of area?

This study conducted ANCOVA to examine the immediate learning achievements (preserving area, indirect comparison of area, comparison of area using a nonstandard unit, and measuring area with multiplication) of the groups using virtual and physical manipulatives. The tests for homogeneity of covariate regression coefficients for various types of manipulatives were not significant for any of the four subscales, suggesting that a common regression coefficient was appropriate for the covariance portion of the analysis. Therefore, the data of the four subscales were appropriate for further parametric analysis. The ANCOVA results indicate that students in the virtual group performed better than those in the physical group in the following three concepts of area in the immediate posttest (see Table 3): preserving area (F = 5.657, p = .021, partial η^2 = 9.2%), comparison of area using a nonstandard unit (F = 6.268, p =. 015, partial η^2 = 10.1%), and measuring area using multiplication (F = 6.989, p

= .011, partial η^2 = 11.1%). Although no significant difference in indirect area comparison was found between the experiment group and the control group (F = 3.584, p = .064), a trend was observed in which students in the virtual manipulative group had a higher mean score (23.51) on indirect comparison of area than those in the physical manipulative group (22.02).

Table 3. Summary of adjusted group means of immediate learning outcomes of participants

Subscales	The virtual group (n = 28)	The physical group (n = 31)
Preserving area	21.10 (4.81)	17.94 (6.98)
Indirect comparison of area	23.51 (2.69)	22.02 (5.73)
Comparison of area using a nonstandard unit	22.97 (3.00)	20.66 (5.88)
Measuring the area using multiplication	20.36 (4.01)	17.96 (6.66)

Research Question 2: Does the use of Magic Board have a more positive effect than physical manipulatives on the retention performance of students studying concepts of area?

This study conducted ANCOVA to examine the retention learning achievements (preserving area, indirect comparison of area, comparison of area using a nonstandard unit, and measuring area with multiplication) of the groups using virtual and physical manipulatives. The tests for homogeneity of covariate regression coefficients for various types of manipulatives were not significant for any of the four subscales, suggesting that a common regression coefficient was appropriate for the covariance portion of the analysis. Therefore, the data of the four subscales were appropriate for further parametric analysis. ANCOVA results indicate that students in the virtual group outperformed those in the physical group for all four concepts of area on the retention posttest (see Table 4): preserving area (F = 10.323, p =.002, η^2 = 15.6%), indirect comparison of area (F = 4.908, p =. 031, η^2 = 8.0%), comparison of area using a nonstandard unit (F = 13.021, p = .001, η^2 = 18.9%), and measuring area using multiplication (F = 13.510, p =. 001, η^2 = 19.4%).

Table 4. Summary of adjusted group means of retention learning outcomes of participants

Subscales	The virtual group (n = 28)	The physical group (n = 31)
Preserving area	20.97 (5.18)	17.19 (5.40)
Indirect comparison of area	22.98 (3.34)	19.96 (6.78)
Comparison of area using a nonstandard unit	23.02 (2.38)	19.18 (6.20)
Measuring the area using multiplication	19.91 (4.02)	16.27 (6.14)

Discussion

This study developed teaching materials for Magic Board and compared immediate learning and retention performance between the use of Magic Board and physical manipulatives. The use of Magic Board has a more immediate positive effect than physical manipulatives in three of the four indicators of student performance when studying concepts of area (preserving area, comparison of area using a nonstandard unit, and measuring area using multiplication). The use of Magic Board has a more positive retention effect than physical manipulatives in all of the four indicators of student performance when studying concepts of area (preserving area, indirect comparison of area, comparison of area using a nonstandard unit, and measuring area using multiplication). The results are not consistent with those of Olkun's study (2003), indicating that solving geometric puzzles using manipulatives, both virtual and physical, has a positive effect on geometric reasoning with regard to two-dimensional geometric shapes, particularly in spatial tasks. However, the overall difference between the virtual and physical groups was not statistically significant. One explanation for the results in Olkun's study is that the functions of physical and virtual versions of tangrams were nearly the same for students learning 2D geometry. However, the virtual group gained more from intervention than did the physical group in our study. This suggests that virtual manipulatives might be more appropriate for studying aspects of 2D geometry. Students did not learn more about planar geometry by touching physical representations. Alternatively, it may occasionally be preferable to use a virtual manipulative instead.

Based on teacher observations and class journals, possible explanations for the significant gains in the immediate learning and retention among students in the virtual manipulative group are as follows. First, for preserving area, students in the virtual group could move objects to a new position and still see the shape of the objects in the original position; however, those in the physical group were unable to do this. Second, virtual tools could record the processes of comparing area indirectly, but physical tools did not provide this function. Third, the virtual group could easily identify the quantity and shape of nonstandard units used for the comparison of area; however, the physical group could not always clearly identify the quantity or shape of nonstandard units because parts of the objects were often covered by their hands. Finally, students in the virtual group were able to cover the object using a row of squares or a column of squares for measuring area using multiplication; however, students in the physical group could only cover the object with one square at a time.

Magic Board provides a good layout for presenting materials, such that learners can pay more attention to learning concepts of area. Presenting teaching materials using physical manipulatives tends not to be as clear, which prevents students from tracing the processes involved in moving objects. Teachers can use Magic Board to quickly and easily pose problems, which gives students in the virtual group more time to discuss the main concepts. Teachers in the physical group have to spend more time using physical manipulatives to pose problems, which reduces the time available for discussion of the teaching materials. Yuan, Lee, and Wang (2010) observed that students perform active thinking and engage in more discussion in a virtual manipulative environment, which may explain the immediate learning and retention performance of using Magic Board. Although no significant difference in indirect area comparison of the immediate posttest was found between the virtual group and physical group, students in the virtual group outperformed those in the physical group for indirect area comparison on the retention posttest. One possible reason for the inconsistent results may be that students in the virtual group had actually absorbed and understood the materials rather than merely relied on short-term memory as compared with those in the physical group.

This research has a number of implications. First, modifying the physical environment to enable students to easily observe the processes involved in moving objects might result in outcomes similar to those demonstrated using Magic Board. In other words, whether the materials are virtual or physical might make little difference as long as the method of instruction is preserved (Yuan, Lee, & Wang, 2010). This is an interesting issue for further investigation. Second, the sample size of this study was small; therefore, generalized findings may be limited to similar samples, and are not necessarily applicable to other groups of learners with diverse educational or cultural backgrounds. Third, the characteristics of the course on Concepts of Area differ considerably from those of other domains such as biology or social sciences. Thus, the conclusions of this study cannot be generalized to other disciplines. Finally, prior experience in areas such as mathematics epistemology or computer self-efficacy may influence learning outcomes when using virtual or physical manipulatives. Future studies would need to deeply examine the role of prior experience on learning with virtual manipulatives.

Magic Board has the potential to improve the learning outcomes (immediate learning and retention) of students in the construction of mathematics knowledge. Magic Board is a tool that provides many of the common virtual manipulatives found in mathematics textbooks, without providing any instruction in mathematical concepts. Therefore, teachers must pay more attention to the instructional design to apply virtual manipulatives appropriately. In the future, educators in mathematics should select mathematical topics that are difficult to present using traditional instruction methods and adapt these lessons to Magic Board to help students learn more effectively and efficiently.

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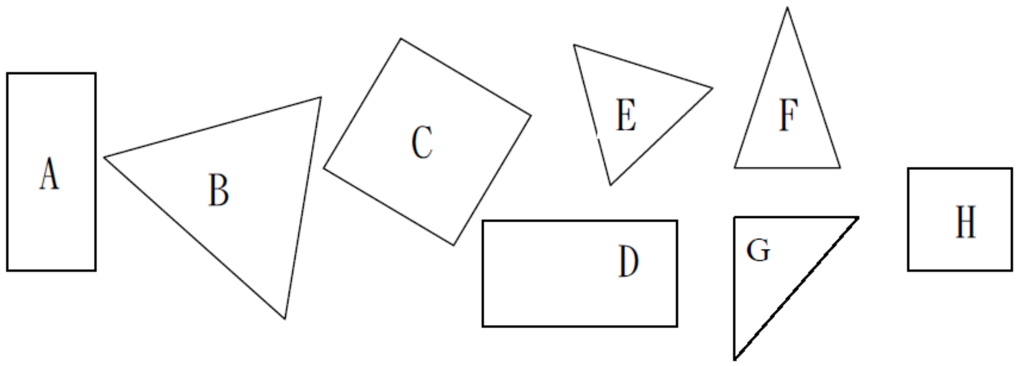
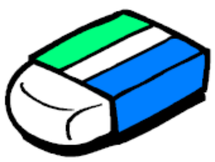
Appendix 1. Sample items of BACT

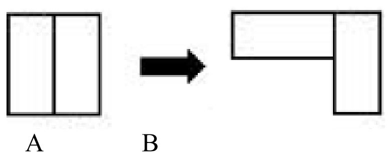
***Identifying and classifying simple 2D geometric figures**
Please see the figures below and measure them with a ruler. Which figure is a square? (1) CDH (2) BEFG (3) CH (4) H



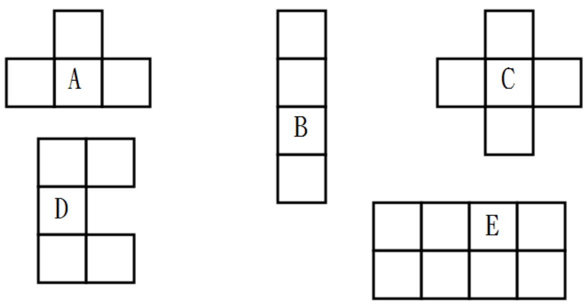
***Cultivating a sense of numbers, length, and estimation**
What could be the length of a normal eraser? (1) 5cm (2) 1m (3) 5m (4) 10m



***Preserving the concept of area**
Cut figure A along the straight line and combine the two parts of A into figure B. If the area of A is 4 cm^2, what is the area of B? (1)3 cm^2 (2) 4 cm^2 (3) 5 cm^2 (4) 6 cm^2



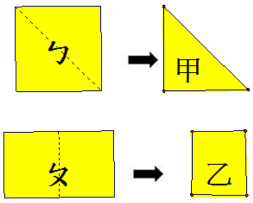
***Comparing the area of various shapes**
Which of the following figures has the largest area? (1) B (2) C (3) D (4)E



Appendix 2 Sample items of ACT

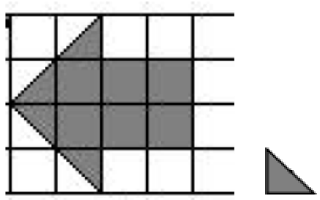
***Preserving area**

The area of figure ㄅ is equal to the area of figure ㄆ. Fold up figures ㄅ and ㄆ to obtain figures 甲 and 乙, respectively. Which statement is true for the area of 甲 and 乙? (1) 甲>乙 (2) 甲=乙 (3) 甲<乙



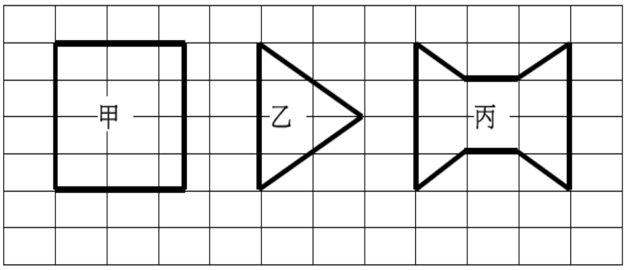
***Comparison of area using a nonstandard unit**

How many of the gray triangles do we need to completely cover the geometric shape on the left? (1) 10 (2) 12 (3) 14 (4) 16



***Indirect comparison of area**

The area of 乙 is 2 cm^2. Which of the following statement is true? (1)The area of 甲 is 7 cm^2 (2) The area of 丙 is 5 cm^2 (3) The area of 甲 is 6 cm^2 (4) The area of 丙 is 4 cm^2



***Measuring the area using multiplication**

Please see the figure below. Which of the following statements is true? (1) The area of ● is 45 (2) The area of ◆ is 31 (3) The area of ★ is 24 (4) The area of ★ >The area of ◆ >The area of ●