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Conceptualizing “Homework” in Flipped Mathematics Classes

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ABSTRACT

Flipped instruction is becoming more common in the United States, particularly in mathematics classes. One of the defining characteristics of this increasingly popular instructional format is the homework teachers assign. In contrast to traditional mathematics classes in which homework consists of problem sets, homework in flipped classes often takes the form of an instructional video. This paper presents a framework for flipped mathematics homework that categorizes types of homework and also draws on technology literature and mathematics education literature to discern quality for each type. For each category of homework, we provide illustrative examples from a study of flipped mathematics implementations. We show how the quality of instructional videos can vary and that, in addition to instructional videos, teachers also assigned videos to motivate subsequent in-class work on mathematics tasks. We close with directions for future work.

Keywords

Flipped instruction, Mathematics education, Teaching, Homework, Multimedia

Introduction

Within flipped instruction, teachers invert or “flip” the settings in which teacher lecture and student practice occur. In the last few years, teachers’ implementation of flipped instruction has increased dramatically. Flipped instruction has now reached the point where more than two-thirds of teachers in the United States report flipping a lesson, if not an entire course (Smith, 2014). Although flipped instruction originated in the context of university-level sciences (Mazur, 1991), it has become especially popular in the context of secondary and post-secondary mathematics (Moore, Gillett, & Steele, 2014; Zack et al., 2015). The increase in flipped mathematics courses is perhaps due to the recent proliferation of online mathematics instructional videos (e.g., Khan Academy) and the longstanding tradition, particularly in the United States, of mathematics instruction characterized by lecture and exposition (Stigler & Hiebert, 1999). This lecture-based approach translates well to video-based delivery. Further, the more frequent use of lectures at the secondary and post-secondary levels than at the elementary level (Banilower et al., 2013) corresponds with greater utilization of flipped instruction in those settings (Yarbro, Arfstrom, McKnight, & McKnight, 2014). Thus, secondary and post-secondary mathematics classes in the United States constitutes a ripe site for empirical investigations of flipped instruction.

The potential innovative power of flipped instruction can stem from videos themselves, from a different use of class time, or from a synergy between the two. Because research on flipped instruction is still limited (Uzunboylu & Karagozlu, 2015), we focus on the most defining component of flipped instruction: the instructional video. Teachers in flipped classrooms typically assign the instructional videos for homework. Thus, the videos replace the problem sets that are the typical form of homework in mathematics. We present a framework for flipped mathematics homework that categorizes types. We also draw on technology literature and mathematics education literature to discern quality for each type.

Literature review

Much of the existing literature related to flipped instruction involves expositional descriptions of how authors intend to implement it (e.g., Bergmann & Sams, 2012) but does not involve empirical, third-party observations of those implementations. In mathematics education, research is emerging but the early studies tend to focus on small numbers of flipped classes and on outcomes rather than on the nuances of the implementations. For example, Clark (2015) examined two flipped algebra classes and compared their performance on unit content tests and their survey responses to those of students in non-flipped algebra classes. Clark found no significant differences with respect to performance and only moderate differences in the survey results, with slight preference indicated toward flipped instruction. This study, however, did not give attention to the specific ways in which flipped instruction was implemented. Further, the study did not account for the fact that other implementations of flipped instruction could vary widely from Clark’s. Similarly, DeSantis and colleagues (2015) compared one instance of flipped geometry instruction with non-flipped instruction and found no significant differences with regard to mathematical performance. They did find a slight difference in student

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opinions, in this case with students preferring non-flipped instruction. Others (Love, Hodge, Grandgenett, & Swift, 2014; Sahin, Cavlazoglu, & Zeytuncu, 2015) found more positive results for flipped instruction in university mathematics courses. However, as with the previously cited studies, the details and variability of flipped instruction were not accounted for in the study design. This may be because there is not yet a robust framework for flipped instruction in mathematics.

To work toward such a framework, we looked beyond studies of flipped instruction itself and considered work related to individual aspects of flipped instruction. One important aspect is the way class time is used during flipped implementations. However, in this article we focus on another important aspect: the flipped homework that sets the stage for in-class activity. Because flipped homework is typically video or multimedia content, one might develop a framework based on the holistic level of whether the video contains a recorded lecture (with minimal post-production), a voice-over presentation (with audio synchronized to slides or on-screen writing), or a picture-in-picture presentation (with extensive post-production). Ilioudi and colleagues (2013), for example, found that, over three weeks, mathematics learning correlated more positively with recorded lecture videos than with voice-over lectures.

Although this format-focused approach is worthwhile, we sought a framework that incorporated the full instructional triangle—teacher, students, and content (Cohen, Raudenbush, & Ball, 2003)—with respect to video or multimedia homework. The instructional triangle highlights that teaching is not just what teachers do, but what they do with students in relation to content. The typical environment for this is in a classroom, but the triangle also provides a way to think about instruction in a home environment with respect to flipped videos or multimedia. The videos contain content and the teacher is present, either virtually by being a voice in the video or indirectly by choosing the video to assign. The students are also present, either watching the video or possibly interacting with it.

To consider each of these facets in more detail, we viewed the content through the lens of the Mathematical Quality of Instruction (MQI) instrument (Learning Mathematics for Teaching Project, 2011). The MQI instrument provides a well-established way to consider the coherence and richness of the expressed ideas. The MQI utilizes a coding framework based on presence/absence and appropriateness/inappropriateness of several features of mathematics instruction. For instructional videos, we found that the MQI's "richness and development of the mathematics," "language," and "unmitigated mathematical errors" sub-categories were of particular relevance because the other sub-categories (e.g., "responding to students") do not relate directly to instructional videos or multimedia assigned as homework. Because the MQI was applied to video-recorded lessons, the application to flipped lecture videos seems an appropriate use of the instrument.

For the teacher, we chose to consider some of the decisions the teacher made (consciously or unconsciously) in crafting the video. We used Clark and Mayer's (2008) six principles of design for instructional materials with embedded multimedia content. The *multimedia principle* states that video developers should wisely and judiciously select and add graphics to text. The *contiguity principle* states that relevant text should appear near those graphics. The *modality principle* states that graphics should be explained with audio. Relatedly, the *redundancy principle* states that audio that simply reads written text aloud should not be included. The *coherence principle* states that graphics and audio should be used only if pertinent. Finally, the *personalization principle* advises that developers use a conversational tone when possible. Taken together, these principles can be used to assess design aspects of videos or multimedia assigned as flipped homework. They are analogous to many of the choices a teacher might make in a classroom setting (e.g., what to write on the board and when, tone and formality of speech).

Finally, we considered the students with respect to their opportunities for interaction. What is the extent to which the flipped homework incorporates *interactive digital elements*, such as embedded questions or other features that require students to interact rather than only watch or listen? Within mathematics education specifically, Moyer-Packenham and Westenskow (2013) defined virtual manipulatives as interactive, dynamic representations that may support students in developing mathematical understanding. For example, teachers might provide students with applets featuring interactive graphs that allow students to see the impact changes to coefficients have on the graph of a particular function. Moyer-Packenham and Westenskow's (2013) review of over 60 research reports found virtual manipulatives to have potentially large positive effects on students' mathematical learning, when used in conjunction with well-designed curricula. They concluded that the use of virtual manipulatives provides students with opportunities to engage in mathematics that differs from those presented by paper-and-pencil activities. These findings indicate virtual manipulatives may be worth including in a framework for flipped mathematics homework. Other opportunities for interaction might be more general, such as questions or hints embedded in the videos.

The literature above relates to videos as a means for presenting mathematical content. A flipped mathematics class, however, may also incorporate videos that are not lectures. Instead, videos may be used to motivate ideas that will be subsequently explored in class. Jackson and colleagues (2013) highlighted in general the importance of setting up a mathematical task for students. In the case of flipped mathematics classrooms, the set up might occur at least in part through the homework video. In a later section, when we present our coding scheme for homework videos, we further elaborate the ways in which we conceptualized distinctions among homework that is assigned to set up subsequent mathematics activity.

Overall, there is a large amount of existing scholarship that relates to particular aspects of flipped homework but there are not yet accepted frameworks that coordinate these aspects in coherent ways. The purpose of this article is to describe a framework of flipped mathematics homework and illustrate it with data from a project investigating the implementation of flipped mathematics instruction at the secondary and post-secondary levels.

Method

We use the term homework to describe the work that is assigned for students to complete at home. In flipped instruction, a lecture or expository video or multimedia presentation is typically assigned to be viewed as homework. In our analysis, we considered all manner of video or multimedia presentations (e.g., combinations of animations, audio, or text) the teachers assigned to students. However, in this study the assignments most commonly consisted of video recordings. Therefore, we will refer to these assignments as "video" for the remainder of the paper.

We examined four U.S. teachers' classrooms—Dr. Moore's college-level calculus class, Ms. Schaefer's college-level college algebra class, Mr. Forrest's high-school level pre-calculus class, and Ms. Temple's eighth grade mathematics class. Dr. Moore, Ms. Schaefer, and Ms. Temple had chosen to flip nearly all of their lessons, whereas Mr. Forrest continued to use a traditional instructional model but included flipped lessons occasionally. For each teacher we observed several lessons, collected copies of all materials (both in class and at home) associated with those lessons, and conducted pre- and post-lesson teacher interviews. We also collected survey data from the teachers about their implementation and perspective on flipped instruction.

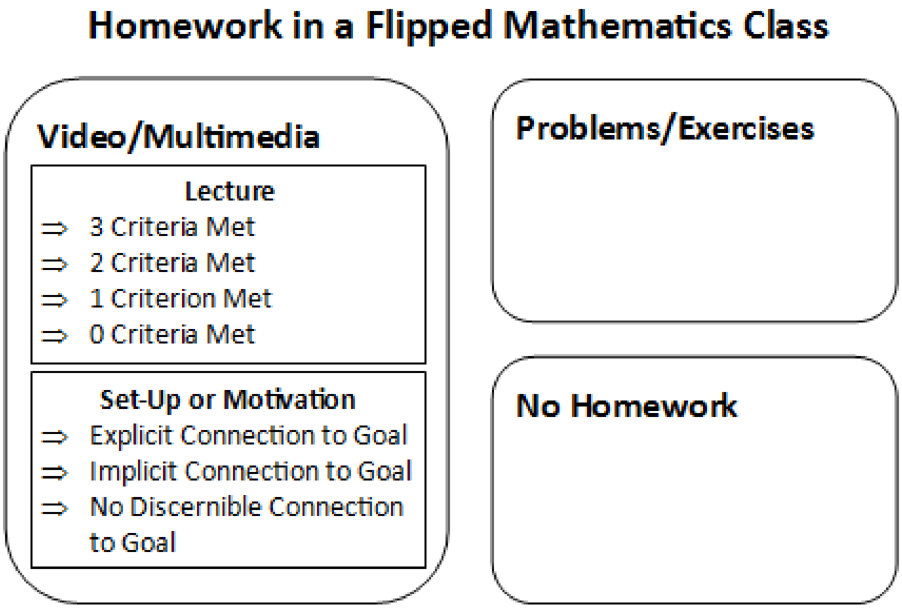


Figure 1. A framework for "homework" in flipped mathematics classrooms

For this particular study, we conducted a two-phase analytic process, constituting our framework for flipped mathematics homework. For the first phase, we categorized the homework according to the teacher's purpose for selecting the video—*lecture* or *set-up/motivation*. We considered these two categories separately because the goal for each differed. The goal for a lecture video (including worked examples) is to deliver content or demonstrate a process. In contrast, a set-up/motivation video is not created for the purpose of content delivery but to establish a situation or context for subsequent mathematical activity in class. Because we wanted a framework that captured the purpose and quality of the homework, we decided to separate these two categories.

This is particularly important because the field of mathematics education has established differing criteria for effective teaching practice for each of these purposes.

We considered not only the videos but also teacher interview and survey data in determining the purpose of the assigned videos. For a particular lesson, it is also possible that the teacher assigns *no homework* at all. It is also possible that a teacher assigns a set of problems or exercises to be completed individually by the students—that is, a typical non-flipped homework assignment. We recognize that teachers using flipped instruction may not flip every lesson. Thus, the inclusion of these additional categories allows us to capture differences between a teacher who flips 100% of their lessons and a teacher who flips many lessons but still incorporates some non-flipped homework assignments or perhaps no homework assignments. Figure 1 contains all of these possibilities for homework in general. However, we focus the remainder of this paper on the video homework because this is characteristic of flipped lessons specifically.

For the second phase of analysis, we assessed the quality of each video in its respective category. This entailed three members of the research team individually coding each video. The full team, five members, then met to resolve any disagreements that resulted from the individual coding. This aspect of our framework is important because it recognizes that the way in which one flips (e.g., the quality of the videos) may be more important than simply whether one flips. Our operationalization of the notion of quality is described in the following sections, wherein we provide illustrative examples of the various types of homework in our framework.

Lecture videos

Each video was coded as either a lecture video or a set-up/motivation video. In mathematics, these lecture videos typically include explanations of mathematical terms or ideas, justifications of mathematical propositions, or worked examples of mathematics problems. They may be created by the teacher or drawn from an existing source such as Khan Academy or educational YouTube channels. For example, a video of a teacher explaining the idea of the y-intercept of linear functions would fall into our category of lecture videos, as would an applet with accompanying text that demonstrates how to solve a certain type of algebra problem. This type of video/multimedia is prevalent in secondary mathematics. This may be due to these videos' alignment with traditional notions of mathematics instruction as involving a teacher explaining ideas to students and showing them what they need to be able to do (Smith, 1996; Stigler & Hiebert, 1999).

Determining the quality

To assess the quality of lecture videos we developed a coding scheme based on the literature reviewed above, particularly portions of the Mathematics Quality of Instruction (MQI) instrument (Learning Mathematics for Teaching Project, 2011), Clark and Mayer (2008), and Moyer-Packenham and Westenskow (2013). Thus, our framework includes three key criteria for high quality lecture video in mathematics:

- it is coded as present-appropriate in the "richness and development of the mathematics" and "language" sub-categories of the MQI and coded as "not present" in the "unmitigated mathematical errors" sub-category of the MQI;
- it adheres to at least five of the six principles of digital material design (Clark & Mayer, 2008); and
- it incorporates virtual manipulatives (Moyer-Packenham & Westenskow, 2013), dynamic representations of mathematical concepts relevant to the goals of the lesson, or digital interactive features (e.g., quizzes, applets, discussion boards).

Note that these criteria are conceptually separate, not integrated, but it is worthwhile to consider them together because they all contribute to the instructional triangle (Cohen, Raudenbush, & Ball, 2003).

A lecture video can satisfy zero, one, two, or all three criteria. For example, a video could violate most of the digital design principles, not incorporate a virtual manipulative, but still have richness and development of mathematics and appropriate language as defined in the MQI. Although coding was based primarily on the video/multimedia itself, future video analyses may require examination of subsequent lesson observation to inform the coding. This subsequent analysis could uncover things such as whether a mathematical error in a video is addressed explicitly by the teacher in class. In the following sections we provide examples from mathematics classes of lecture videos that satisfy different criteria.

Example of video satisfying one criterion

This first example video is from a flipped, college algebra class taught at a community college by Ms. Schaefer. For homework, Ms. Schaefer typically created her own video lectures using Explain Everything (Version 3.21, available from http://itunes.apple.com), a screen casting application for iPads. Explain Everything allows the user to use the iPad as a whiteboard while simultaneously capturing the user's speech in a single video. Ms. Schaefer stated that she did not script the videos prior to recording them. She also noted that the videos took a lot of time to develop. Ms. Schaefer described her expectation for students as they watched the videos.

> Well and I told them it, it's basically as though I were talking to you. If you were in class, what would you be doing? Taking notes, writing things down, important things that I say, same concept. It is just instead of me talking now, I'm talking at home at you. So that's, that's really what we're looking to do there.

We selected a lecture video Ms. Schaefer created to explain how to graph rational functions. This video is illustrative of a typical lecture video in Ms. Schaefer's classroom (Figure 2).

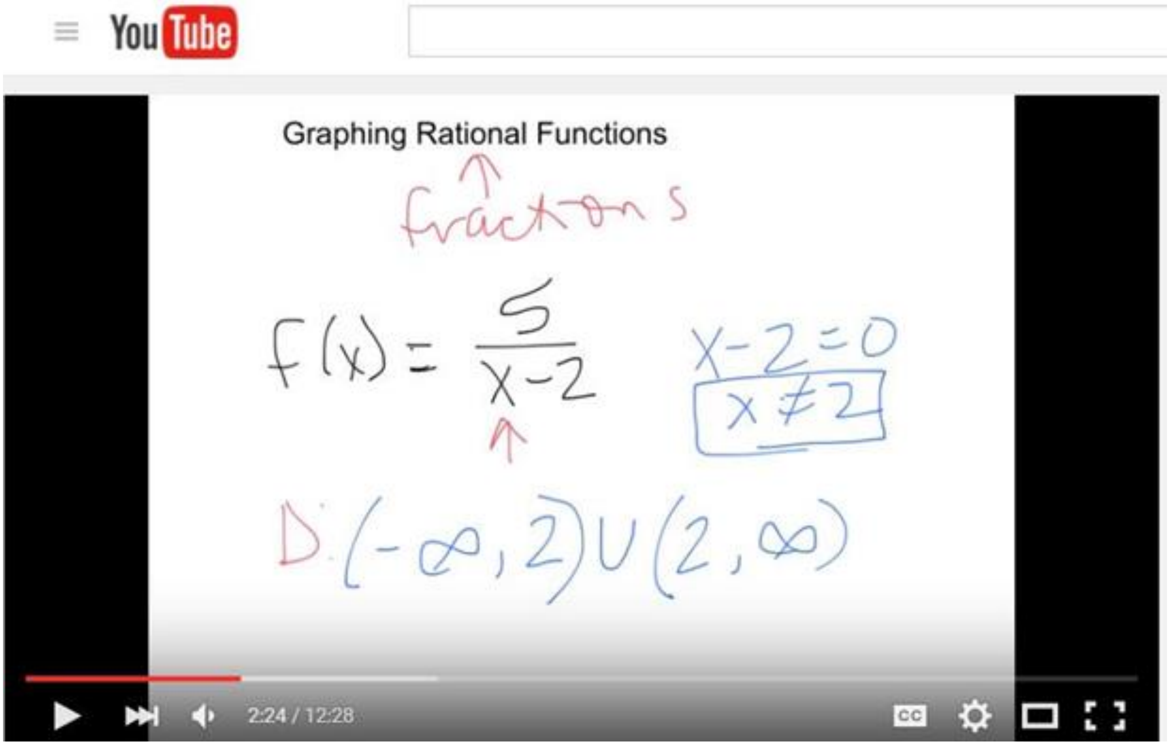


Figure 2. A screenshot from Ms. Schaefer's video

In this 12-and-a-half-minute video, Ms. Schaefer began the video by defining rational functions as those that are written as a fraction in which the variable *x* is in the denominator. Then, Ms. Schaefer explained how to determine the domain of a rational function and then defined asymptotes as imaginary lines that graphs approach but may not cross. For the remainder of the video, Ms. Schaefer described each of the three types of asymptotes (vertical, horizontal and slant (oblique)) and provided examples of rational functions that included each type. During the solution process she used algebraic representations without accompanying graphs or other representations. Although the video focused on graphing rational functions, the sole graph included in the video was a depiction of a vertical asymptote absent the accompanying curve for the rational function. At several points while she worked through examples of how to determine asymptotes she encouraged students to pause the video and check their answers. She took advantage of the multi-colored pen feature of the Explain Everything application to highlight important points.

We assessed Ms. Schaefer's video as meeting one of the quality criteria for lecture videos—Clark and Mayer's (2008) design principles for digital materials. Ms. Schaefer kept a conversational tone throughout much of the video (personalization principle) and the ways in which she incorporated text and audio satisfied the multimedia, contiguity, coherence, and modality principles. She did not meet the redundancy principle because there were a number of instances in which she read aloud text on the screen. Although the video did not have mathematical errors and made use of proper mathematical terminology, the richness and development of the mathematical idea was not highly rated. Therefore, the video did not meet the third criteria related the mathematical content of the lecture. Further, Ms. Schaefer did not include interactive features or virtual manipulatives in her video and so did not meet the second criteria for interactive features.

Example of video satisfying two criteria

Dr. Moore taught a flipped calculus I course at a university. Like Ms. Schaefer, Dr. Moore typically developed lecture videos for his students and often consisted solely of an instructional video. Dr. Moore's videos captured his written work with a video camera rather than screencasts (Figure 3). The video we examined focused on the First Derivative Test and provides a typical example of the videos Dr. Moore created.

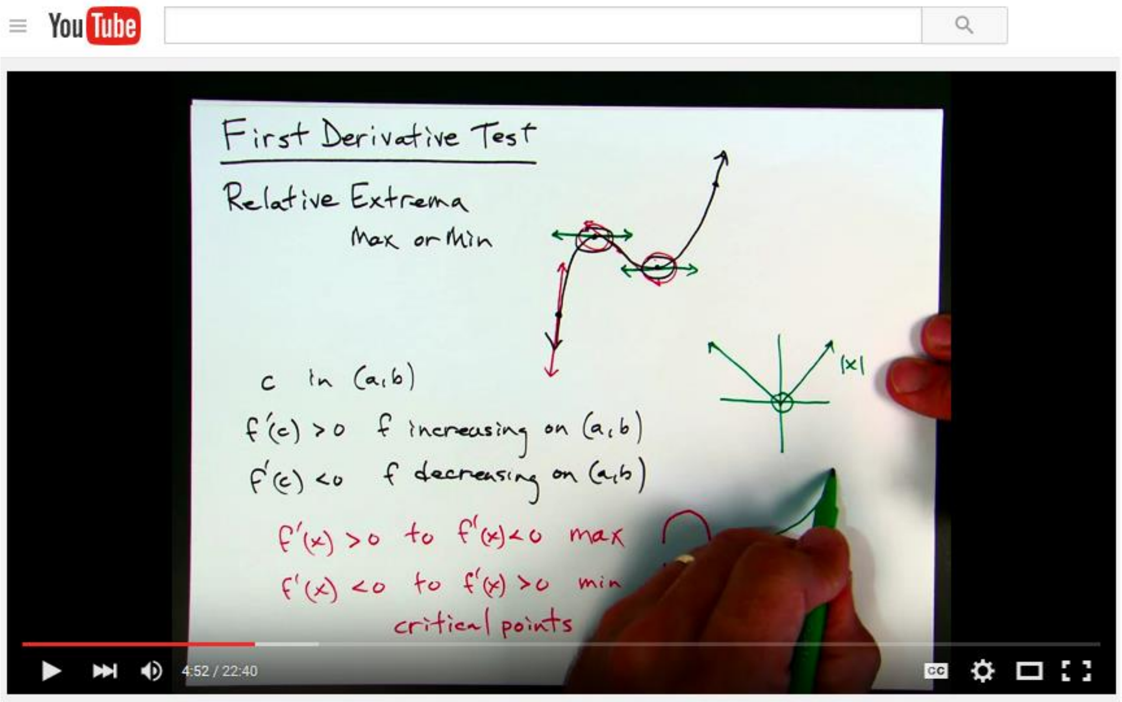


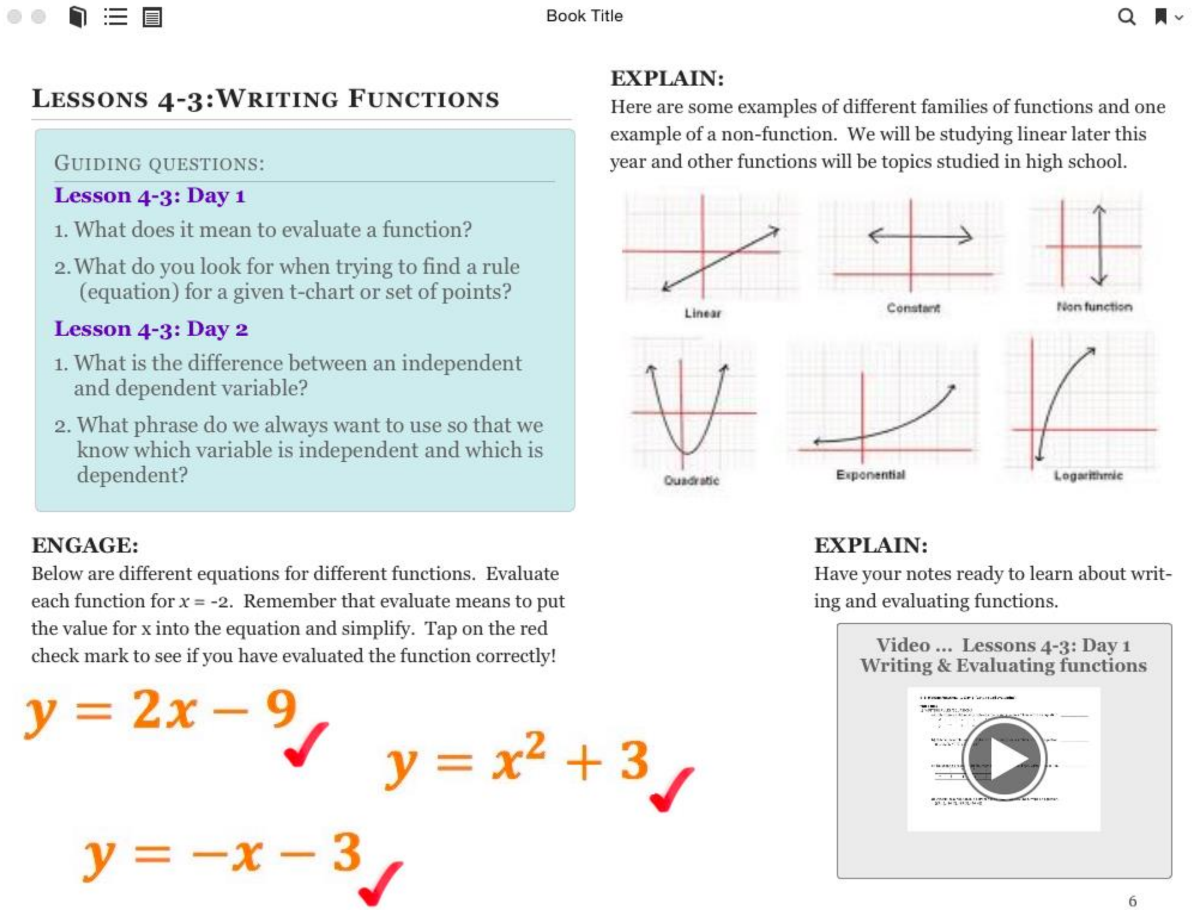
Figure 3. Excerpt from Dr. Moore's video

We analyzed a 22-minute video focused on the First Derivative Test. The video began by identifying and illustrating key terms such as relative extrema. Throughout the video, Dr. Moore referenced events from in-class sessions and used different colors and drawings to highlight important points. Dr. Moore included a number of examples to illustrate the concepts he discussed. For example, when discussing tangent lines with slopes of zero, he drew two such examples on the graph (Figure 3). After discussing the purpose for using the First Derivative Test, Dr. Moore gave several worked examples of how to use the test.

We determined that Dr. Moore's video met two of the quality criteria. Like Ms. Schaefer, Dr. Moore's video adhered to the multimedia, contiguity, modality, and coherence principles. Although the video did not generally use a conversational tone (personalization principle), Dr. Moore avoided much of the redundancy of reading written text aloud. Thus, the five principles that were addressed were sufficient to meet the first criteria for high quality videos. Dr. Moore's video was free of mathematical errors and the mathematical language was appropriate. Further, in contrast to Ms. Schaefer's video, Dr. Moore included a rich mathematical explanation of why the First Derivative Test works. Therefore, the video met the third criteria. Like Ms. Schaefer's video, Dr. Moore's video did not include any virtual manipulatives or interactive features and therefore did not meet the second criteria.

Example of video satisfying three criteria

Our final example comes from Ms. Temple's flipped eighth grade mathematics class. Ms. Temple created her lecture videos in an iBook format. This format allowed her to not only provide students with videos but also include text, examples, practice problems, and embedded quizzes. The homework we considered was from a chapter on scatterplots. In assessing Ms. Temple's homework, we considered all the features of the iBook shown in Figure 4. In addition to the iBook, Ms. Temple provided her students with a set of guided notes to complete. The guided notes included references to particular pages of the textbook, had questions for the students to answer, and indicated when and how students were to engage with the iBook. Thus, students used the notes to guide them through the entirety of the homework which drew on both the iBook and textbook. Despite the variety of resources Ms. Temple provided students for homework, we categorized this assignment as a lecture video because her purpose was to instruct students on the topic of scatterplots.



LESSONS 4-3:WRITING FUNCTIONS

GUIDING QUESTIONS:

Lesson 4-3: Day 1

1. What does it mean to evaluate a function?
2. What do you look for when trying to find a rule (equation) for a given t-chart or set of points?

Lesson 4-3: Day 2

1. What is the difference between an independent and dependent variable?
2. What phrase do we always want to use so that we know which variable is independent and which is dependent?

EXPLAIN:

Here are some examples of different families of functions and one example of a non-function. We will be studying linear later this year and other functions will be topics studied in high school.

ENGAGE:

Below are different equations for different functions. Evaluate each function for $x = -2$. Remember that evaluate means to put the value for x into the equation and simplify. Tap on the red check mark to see if you have evaluated the function correctly!

$y = 2x - 9$ ✓

$y = x^2 + 3$ ✓

$y = -x - 3$ ✓

EXPLAIN:

Have your notes ready to learn about writing and evaluating functions.

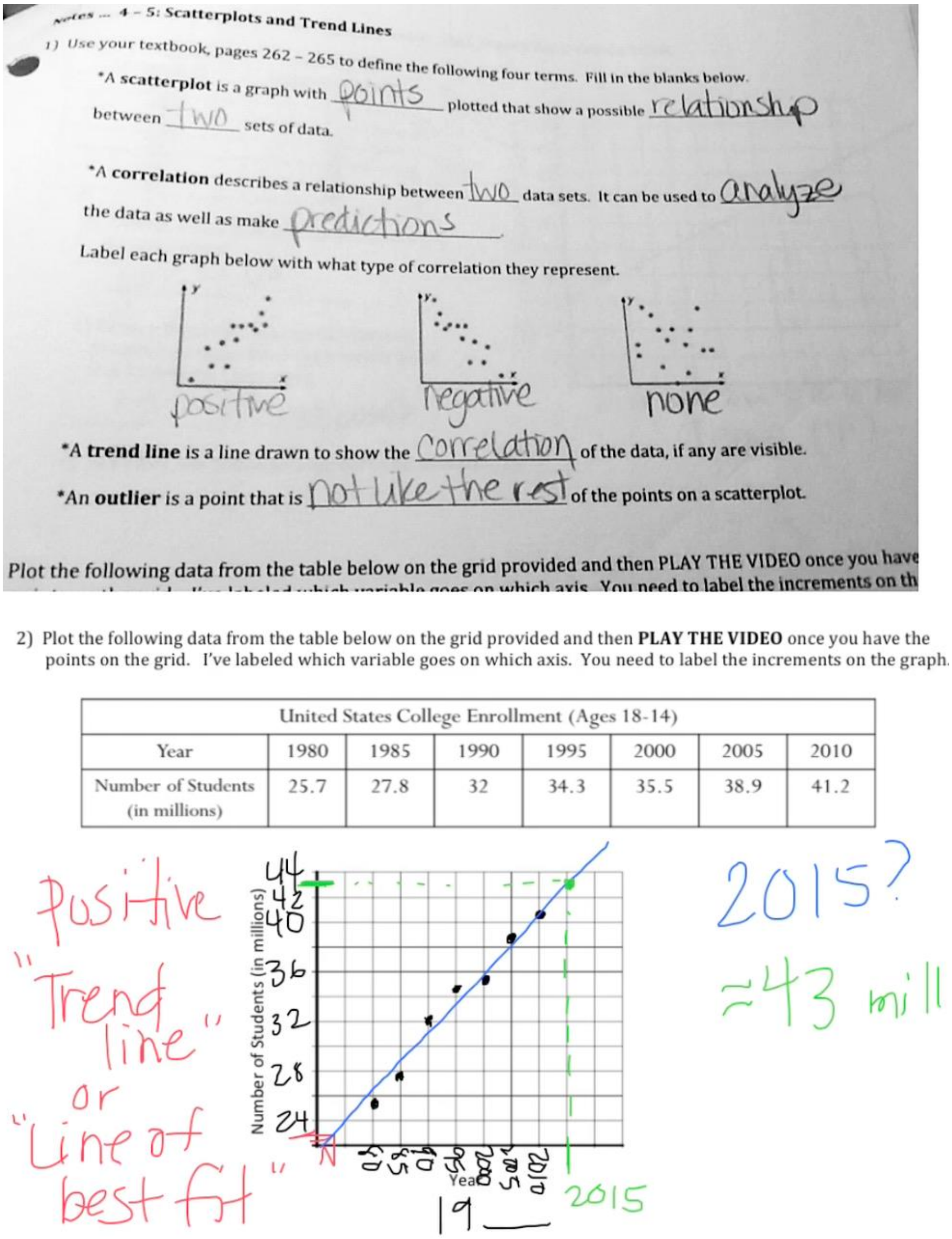
Video ... Lessons 4-3: Day 1
Writing & Evaluating functions

6

Figure 4. A screenshot from Ms. Temple's homework assignment

The first section of the iBook chapter included a set of five guiding questions for the students to consider as they worked through the homework. Ms. Temple also included examples of scatterplots with positive, negative, and no correlation. The students were then to watch a video. Ms. Temple created a six-and-a-half-minute video to further explain what a scatterplot is and how to create one. The video featured screencasts of Ms. Temple's written work on an iPad with the accompanying audio. The video began by providing students with the correct solutions to the questions on the guided notes and encouraged students to pause the video to check their answers (Figure 5). Then, Ms. Temple provided a worked example of how to create a scatterplot and a line of best fit. Throughout the example, she emphasized conventions for labeling the scatterplot's axes and identified common student errors. She then provided a problem for students to complete and asked students to pause the video to complete it. She closed the video by providing the solution for the practice problem. Following the video, the students were to complete a set of review questions on the iBook to check their understanding. The final component of the homework was an online quiz for the students to complete, the results of which Ms. Temple could retrieve online.

We assessed Ms. Temple's homework as meeting all three of the criteria for high quality lecture videos. Ms. Temple's video, like the videos from Ms. Schaefer and Dr. Moore, met the first criteria for design principles of digital media. Unlike the others, however, Ms. Temple's video included all six principles. For example, she avoided reading text that was provided in graphic form (redundancy principle) and described given graphics with audio (modality principle). Like Dr. Moore, Ms. Temple used proper mathematical language and did not have mathematical errors in her video. Although video did not include a rich development of the mathematical idea, the textbook section students were instructed to read did. Thus, the video fully addressed the third criteria. Finally, in contrast to the other two videos, Ms. Temple's iBook did include interactive features (the review questions and practice problems), satisfying the second criteria.



Notes ... 4 – 5: **Scatterplots and Trend Lines**

1) Use your textbook, pages 262 – 265 to define the following four terms. Fill in the blanks below.

*A **scatterplot** is a graph with points plotted that show a possible relationship between two sets of data.

*A **correlation** describes a relationship between two data sets. It can be used to analyze the data as well as make predictions

Label each graph below with what type of correlation they represent.

positive negative none

*A **trend line** is a line drawn to show the correlation of the data, if any are visible.

*An **outlier** is a point that is not like the rest of the points on a scatterplot.

Plot the following data from the table below on the grid provided and then PLAY THE VIDEO once you have

2) Plot the following data from the table below on the grid provided and then **PLAY THE VIDEO** once you have the points on the grid. I've labeled which variable goes on which axis. You need to label the increments on the graph.

United States College Enrollment (Ages 18-14)							
Year	1980	1985	1990	1995	2000	2005	2010
Number of Students (in millions)	25.7	27.8	32	34.3	35.5	38.9	41.2

Figure 5. Excerpts from Ms. Temple's video

Summary

In looking across the three video examples we see three different levels of quality. Though both Ms. Temple and Dr. Moore's videos included well developed mathematical explanations, proper use of language, and were free of errors, Ms. Temple's inclusion of interactive features led to the video's classification as higher in quality than Dr. Moore's. These interactive features help to engage learners in mathematical activity rather than passively viewing the videos. We posit that, based on extant literature (e.g., Moyer-Packenham & Westenskow, 2013), the inclusion of these interactive aspects may provide greater opportunities for student learning and so videos with these features can be assessed as higher in quality than those absent them.

In contrast to both Dr. Moore and Ms. Temple’s videos, Ms. Schaefer’s video focused solely on procedures and did not motivate why or how the procedure for determining asymptotes works. This in conjunction with the video’s lack of digital manipulatives or interactive features led to this particular video assessed as lower in quality than the other two videos because it met just one of the criteria. This assessment indicates there may be less opportunities for students to learn and engage in mathematics as they view the video. For example, though students may gain some familiarity with how to determine an asymptote for a particular function, the video does not indicate how or why this procedure is needed. Further, students may finish watching the video and have little understanding of asymptotes as features of a particular function rather than an independent graph. The difference in content and user experiences presented in these examples motivate the need to consider criteria for moving beyond top level categories such as lecture videos into criteria to describe the quality of these videos.

Set-up/Motivation videos

Not all videos assigned as homework in flipped instruction are expository in nature. In some cases, a teacher assigns students a video that poses a mathematical problem or establishes a non-mathematical context motivating the work that will occur during the subsequent class period. We call these *set-up*/*motivation* videos, and they occurred less frequently in our data than lecture videos.

Determining the quality

There are no existing coding schemes specifically designed to identify the effective features of these types of videos in mathematics. There is, however, research on the process of setting up mathematical tasks in class. For example, Jackson and colleagues (2013) examined how teachers use the set-up to establish common language, introduce the context of the task, or review mathematical relationships that will be pertinent. They argued that these set-up practices have implications for subsequent learning opportunities. González and Eli (2015) also studied the tensions between providing enough set-up for students to get started but not so much that it reduces the cognitive demand of the task.

This existing research depends largely on the actions of the teacher and so the analytic approaches do not apply directly to set-up videos themselves. Thus we devised a new coding scheme for assessing the quality of set-up videos. In this scheme, the video is considered together with any questions or activities that are also assigned as part of the homework:

- it *explicitly* prompts a mathematical problem or idea that will be explored or developed as part of the lesson’s learning goals;
- it *implicitly* contains a mathematical problem or idea that will be explored or developed as part of the lesson’s learning goals; or
- it does *not contain* a discernible mathematical problem or idea that will be explored or developed as part of the lesson’s learning goals.

Such videos, by definition, do not contain a full lecture or exposition of mathematical content. However, these codes describe the extent to which they prompt or prepare the students for the mathematical activity that will occur in class. A video is of the explicit type if one can watch the video (and read any associated homework questions, such as what to attend to in the video) and predict with confidence the mathematical learning goal of the subsequent in-class lesson. Such a video may intellectually motivate a purely mathematical question or it may present a mathematical idea in an applied context. In either case, the mathematical idea of interest is explicit and clear. A video of the implicit type contains mathematical ideas that will serve the lesson’s learning goals. In contrast to explicit types, however, the learning goals are not obvious from simply watching the video itself. They may only be clear in retrospect after seeing the subsequent in-class activity. The final type of set-up video is one that does not contain mathematical ideas relevant to the lesson’s learning goals but perhaps only relates to a real-world context. We further illustrate each of these three types of set-up video in the next sub-sections.

Example of a video that implicitly connects to a learning goal

Mr. Forrest was a high school mathematics teacher. His teaching approach involved students being “the drivers of the mathematics” because they “retain more when they ‘discover’ concepts on their own versus having [the teacher] deliver it to them.” Most of his lessons were not flipped but consisted of in-class investigations, followed by independent practice as homework. He did, however, flip some individual lessons. The video we examined came from his pre-calculus class and focused on the use of trigonometric functions to determine unknown components of triangles. The video shows a plane landing at an airport that is just behind a public beach in Sint Maarten (see Figure 6). Mr. Forrest asked the students to watch the video and write down questions they wanted to have answered about the situation.

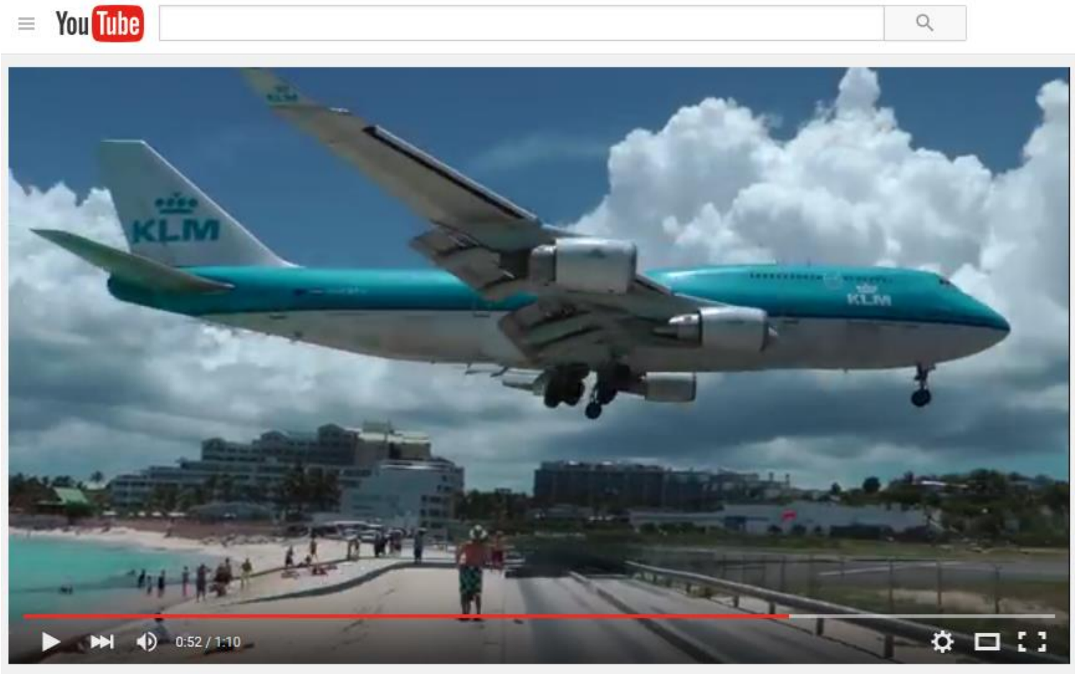


Figure 6. A screenshot from Mr. Forrest's assigned video

In the video, there are many mathematical and non-mathematical phenomena to which the students could attend. Moreover, the directions accompanying the video allowed the students to generate any question rather than focusing them on formulating questions that can be answered with trigonometric functions. Many students did ask questions appropriate to the trigonometric goals of the subsequent in-class work. For example students asked about the height of the plane above the beach, the height of the plane above the chain-link fence, and the distance the man in the hat is from the runway. However, several other students asked questions that did not align with the goals, such as how loud the plane was or how many planes land there each day.

Overall, this video establishes a context for trigonometric work. It implicitly contains mathematical relationships relevant to the lesson goals, such as the angle of descent formed between the runway and the plane's motion. However, Mr. Forrest, not the video, was the one to guide students in identifying the key mathematical relationships. Further, he had to clarify which of the students' many questions were appropriate with regard to the lesson goals. The video also did not explicitly contain the required information needed to answer the questions. Mr. Forrest, for example, directed students to use the internet to look up typical airplane landing angles because it could not be measured accurately from the video.

Other examples

Because of the rarity of set-up videos in our data, we briefly mention other examples drawn from external sources. Meyer (2011) wrote about creating or selecting videos in which the desired mathematical question is clear or even unavoidable. Figure 7 shows an image that aligns with our notion of explicit prompting because a natural question is whether the basketball will go through the hoop. Furthermore, because the video was shot from a fixed position and the relevant features are in view (e.g., the basketball's path, the hoop, the ground), the video lends itself for direct use in the subsequent in-class activity related to quadratic functions.

Finally, a hypothetical example of a video that has no discernible connection to the lesson's mathematical goals would be a video clip from a popular movie assigned as homework because the subsequent lesson will involve analyzing movie box office statistics. In this case, the video is meant to excite students or connect to their natural interest in movies but it does not actually contain any box office statistics or any content that will be central to the lesson's learning goals. In other words, it focuses purely on the context of subsequent in-class activities, not the mathematical ideas. Such a video would be analogous to some of the non-mathematical marginal images that are included in U.S. mathematics textbooks as purely visual elements or as opportunities to connect with students' non-mathematical interests; such marginal images are absent from the textbooks of some other countries, such as Singapore (Erbas, Alacaci, & Bulut, 2012).

Figure 7. An image from an explicit video example

Summary

Set-up/motivational videos can be more or less explicit about the learning goals that follow. Nevertheless, this category of video constitutes a substantial departure from typical non-flipped homework because it is about preparing for mathematical work rather than practicing a skill that has already been taught. These videos, however, constitute only the starting point of the actual set-up to the mathematical work. Mr. Forrest, for instance, still had more set-up work to do in class as he took the students' questions and formulated them into a specific trigonometric task. Teachers may also, to greater or lesser degrees, engage in the kinds of set-up practices that Jackson and colleagues (2013) identified, such as establishing common language. Hence this category may be particularly challenging to analyze in isolation from how it is used subsequently in class.

Conclusion

We presented a framework that distinguishes between various purposes and quality of homework in flipped mathematics classes. This framework provides a basis for design research focused on developing effective materials for flipped instruction in mathematics. Development of flipped materials is already occurring at an increasing rate but is not yet informed by empirical evidence. By establishing shared conceptions of various types of homework and homework quality, researchers can move forward in conducting investigations of how these various forms impact student learning, if at all.

Because this framework was developed from a relatively small data set, future work is needed to understand whether the framework can adequately categorize teachers' homework practices in flipped classrooms. Furthermore, additional work is required to understand whether the proposed quality indicators impact student learning. Thus, although we have developed this framework based on literature on mathematics teaching in non-flipped environments and our sample of flipped classrooms, future work is needed to further refine this framework.

In our ongoing work, we have thus far found that teachers seldom included interactive features in their lecture videos. We have also yet to find videos that included the dynamic virtual manipulatives described by Moyer-Packenham and Westenskow (2013). We posit this may be due to the teachers' lack of familiarity with such tools or the time required to embed interactive features into videos. We also found that, by a wide margin, the teachers we studied more frequently assigned lecture videos than set-up/motivation videos. Further study is needed to understand whether the type of homework has differing impact on student learning and whether the proportions we found were similar to those found in the wider population of teachers.

Additionally, further study is needed to examine the quality criteria we have proposed. In particular, it is not known whether certain criteria are more important than other criteria in terms of the impact homework has on student learning. For example, could the mathematical features of a lecture video have a greater impact on student learning than the digital design principles, or vice versa? Future work might bring members of the educational technology and mathematics education communities together to better understand if integration of the criteria is possible and/or necessary. For example, is there a way to integrate the MQI with features of digital design, rather than consider them separately as we have done? Although these questions remain unanswered, the framework presented provides a first step and common conception from which to devise such studies.

In future investigations of homework in flipped classroom, student behavior is an important mediating factor to consider. Student behavior refers to the extent to which students actually watch the videos, or not, and to students' behaviors while watching the videos (pausing, skipping ahead, multitasking, etc.). Although the work at home can directly influence what occurs in class, it is possible that student behaviors can greatly mitigate this influence. For example, if a high-quality lecture video is assigned but no students watch it, the video is unlikely to have a strong relationship with the in-class activity or the students' learning. Finding ways to measure and categorize these behaviors is crucial for those who wish to examine connections between flipped instruction and student learning outcomes.

Looking beyond homework, it is likely that the in-class implementation of flipped instruction is just as important or more important than the homework. For example, after students watch a set-up video as homework, a teacher might organize an engaging and cognitively-demanding task that builds on the video or might simply ask students to complete a number of low-level tasks loosely associated with the video. These different in-class choices may be more important than the quality of the homework. In other words, the real potential of flipped instruction may not lie in the videos but in what the flipped format allows the teacher and students to do together in the classroom.

Understanding flipped instruction is crucial because although a majority of teachers have implemented this model of instruction, little is known about how teachers are implementing it, it is also unclear what impact flipped instruction may have on student learning. The existing literature base is predominantly expositional or anecdotal and is often written in ways that suggest flipped instruction is a unified teaching model. Our work suggests that flipped instruction varies greatly and this variance must be attended to in order to draw meaningful conclusion. Our framework draws distinctions among homework types that could set the foundation for more nuanced investigations of flipped learning. This may in turn help the research base keep better pace with implementation.

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