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Integrating Dynamic Mathematics Software into Cooperative Learning Environments in Mathematics

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ABSTRACT

The aim of this study was to evaluate the implementation of the cooperative learning model supported with dynamic mathematics software (DMS), that is a reflection of constructivist learning theory in the classroom environment, in the teaching of mathematics. For this purpose, a workshop was conducted with the volunteer teachers on the implementation of the cooperative learning model supported with DMS. Dynamic materials and worksheets suitable for quadratic functions and sequences topics were developed. The effect of implementing the cooperative learning model supported with DMS in the teaching of the quadratic functions and sequences topics on student performance as well as students' views about the model were examined. The study was carried out using an embedded design. The study group consisted of 61 high school students. A quadratic functions knowledge test, a sequences knowledge test, and an open-ended questionnaire were used as data collection tools. The Mann–Whitney test and dependent *t*-test were used for the analysis of quantitative data, while content and descriptive analyses were used for the analysis of qualitative data. As a result of analysis of the data, it was found that the model had a positive effect on student achievement. Moreover, the following students' views were identified: the model enabled better understanding, it visualized and concretized the course, and it created a pleasant and enjoyable learning environment.

Keywords

Dynamic mathematics software, Cooperative learning, GeoGebra, Mathematics learning and teaching

Introduction

Social changes have triggered rapid development. Information and communication technologies (ICTs) have affected human life and have resulted in new opportunities to emerge and new information to be created. These innovations require perspectives and expectations towards mathematics, the use of mathematics in different ways and a review of mathematics instruction. In line with these changes and developments, new problems have occurred in our life each passing day and individuals who have realized the importance and value of mathematics and who have developed their power of thinking are needed more than ever (Turkish Ministry of National Education [TMoNE], 2013). However, as mathematics is more abstract and difficult (Herzig, 2002) when compared to other disciplines (Dienes, 1971; Frenkel, 2013; Sarama & Clements, 2009), it results in difficulties in the teaching and learning of mathematics (Yenilmez & Avcu, 2009).

As most mathematics concepts are abstract, they cause difficulties in the teaching and learning processes. Functions are one of the most important concepts which are difficult to learn, resulting in misconceptions (Ural, 2006) and play both a central and connective role among mathematics topics (Selden & Selden, 1992). As a result of the close relationship between functions and mathematical thinking, the active use of functional thinking for solving problems in different disciplines (Bayazıt & Aksoy, 2013), the important role of learning functions and graphs in understanding mathematics (Kutluca & Baki, 2013) and the difficulties while learning the topic are increasing more and more. Quadratic functions and graphs are special and important states of functions and they are the basic principles of functions for secondary school students (Even, 1990). However, it is known that students have difficulty with quadratic functions and graphs just like with functions (Kutluca & Baki, 2009; Sajka, 2003; Tatar, Okur, & Tuna, 2008; Zazkis, Liljedahl, & Gadowsky, 2003). After the functions topic is taught in the 9th grade in the secondary school mathematics curriculum, students again encounter quadratic functions and graphs in the 10th grade in Turkey. Similarly, in 11th grade, there are topics which mainly involve functions. The sequences topic which is linked to the functions topic is one of the main topics in which students have difficulty in understanding in the 11th grade in Turkey (Akgün & Duru, 2007; Durmuş, 2004; Tatar, Okur, & Tuna, 2008).

Functions, quadratic functions and graphs, sequences, numbers and algebra are generally among the difficult topics to learn. Students continue to meet these topics in the numbers and algebra learning domain of the curriculum in the mathematics courses taught in their undergraduate studies, particularly relating to the knowledge of functions. It is important that these concepts which are used as basic ideas in mathematics courses like advanced analysis and differential equations, should be learned (Bayazıt, 2010). Therefore, it is considered that research studies on these topics which are included in the learning domain of numbers and algebra would be useful to students, mathematics teachers, and mathematics educators.

Boosting mathematical topics which involve abstract concepts with ICT can enable mathematics to be concretized and help concepts to be learned more easily (Baki, 2002). Moreover, students who are actively engaged in the problem-solving process with the use of ICTs, experience different representations of concepts rather than repeated calculations, and work on real mathematics problems (TMoNE, 2013). Open source dynamic mathematics software (DMS) GeoGebra is a freely available and easy-to-use ICT (Diković, 2009; Hohenwarter & Preiner, 2007).

Examining and utilizing ICTs within the scope of suitable learning approaches is as important as choosing the most appropriate tool. The role of technology is not to transfer knowledge but to enable a learner to construct knowledge and make sense of their experiences based on the constructivist learning theory (Tezci & Perkmen, 2013). The cooperative learning model is considered as an important tool which promotes the reflection of constructivist learning theories that regard learning by discovery and learning as a social activity in the classroom environment (De Lisi & Golbeck, 1999). There are many methods used in the cooperative learning model. One of them is the Student Teams Achievement Division (STAD) and it is implemented in many majors such as mathematics, science, social studies and industrial arts at all levels from primary education to higher education (Slavin, 1994). Moreover, because the STAD motivates students, it teaches them to help each other (Slavin, 1987) and as it is practiced easily (Slavin, 1995), it was preferred in this study as GeoGebra. GeoGebra is a suitable ICT software that uses the cooperative learning model based on the constructivist learning theory, particularly for teaching of difficult concepts like quadratic functions and sequences.

The DMS has a positive effect on student achievement and motivation in mathematics. When teachers use this software in mathematics classrooms students increased attention and interest engender better understanding and facilitate conceptual learning (Choi, 2010; Tatar & Zengin, 2016). When the literature about using DMS in learning and teaching of difficult mathematics concepts is examined, it is found that DMS provides positive effects such as an increase in achievement (Ayvaz Reis & Özdemir, 2010; Saha, Ayub, & Tarmizi, 2010), an interesting, visual and concrete learning environment (Tatar & Zengin, 2016), and opportunities for problem-solving. Considering the positive findings about how to integrate this dynamic software in the learning environment, it is very important for educators to evaluate these positive capabilities of DMS. On the other hand, one of the important skills in the 21st century is the inculcation of cooperative skills. Cooperative learning facilitates the development of improved inter-group relations, social acceptance, increased self-esteem, and mathematics achievement. The other contributions of this method are that students improve their problem-solving abilities and the ability to integrate specific knowledge and skills (Slavin, 1995). In this regard, DMS GeoGebra was embedded in cooperative learning in this study for integrating the strengths of both the software and the cooperative learning model. The cooperative learning model supported with DMS was implemented in difficult concepts in mathematics. When the literature is examined, it is found that students have difficulty in understanding about quadratic functions (Kutluca & Baki, 2009; Sajka, 2003; Tatar, Okur, & Tuna, 2008; Zazkis, Liljedahl, & Gadowsky, 2003) and about sequences (Akgün & Duru, 2007; Durmuş 2004; Tatar, Okur, & Tuna, 2008). There are many separate studies about the use of DMS and cooperative learning in mathematics. However, the use of both quadratic functions and sequences in high school level is not found in the literature. The significance of this research is that it may provide a good alternative model for teaching difficult mathematics concepts such as quadratic functions and sequences in high school mathematics. In addition, the other significance of this research is that it may contribute to integrating DMS in the learning environment using the cooperative approach. Therefore, a significant attention to the effectiveness of the model at the high school level of mathematics teaching is crucial. In this context, the research questions are as follows:

- What is the effect of the cooperative learning model supported with DMS on student achievement in mathematics?
- What are the views of students on the use of the cooperative learning model supported with DMS in mathematics?

Method

Both qualitative and quantitative data were gathered and analyzed concurrently or sequentially throughout the research process using an embedded design. One of the purposes of this design is that the qualitative data obtained must support the quantitative data or vice versa. Moreover, this design is suitable for interpreting different research problems with different types of data. The advantage of this design is that it integrates the benefits of both quantitative and qualitative data. The weakness of this design is that the two sets of data may be difficult to compare because each data set addresses different research questions. Furthermore, the data collection is very labor intensive for a single researcher (Creswell, 2012). In the study, the effect of the implementation of the cooperative learning model supported with DMS in the quadratic functions and sequences topics on student achievement was evaluated using the quantitative data and students' views about the model. The embedded design was used in this research study because it provided opportunities to collect data for the different problems of the study using different data collection procedures. There were two experimental groups, one each for sequences and quadratic functions, as well as a control group in this study. The cooperative learning model supported with DMS was implemented with the experimental group for the sequences topics for three weeks. The control group was instructed in sequences only in a traditional method for three weeks. The experimental group for the quadratic functions topic was instructed in the cooperative learning model supported with DMS for two and a half weeks.

Study group

The study group consisted of 61 high school students in Turkey of whom 27 were male and 34 were female students. Participants' ages ranged from 16 to 17 years. The experimental group for the sequences topic, the experimental group for quadratic functions, and the control group consisted of 19, 25, and 17 high school students, respectively. The participants were chosen using the convenience sampling method. This sampling method is preferred because of students' convenient accessibility and proximity to the researchers (Yıldırım & Şimşek, 2011).

Data collection tools

Using the data obtained with different methods increases the reliability and validity of the results obtained (Yıldırım & Şimşek, 2011). Therefore, both qualitative and quantitative data collection tools were used in the research study. In the research, the Sequences Knowledge Test (SKT), the Quadratic Functions Knowledge Test (QFKT), and an open-ended questionnaire were used. The data collection tools used in the study are discussed in detail below.

Knowledge test

The SKT and QFKT were administered as pre-test and post-test. While developing the test, the TMoNE (2011) and TMoNE (2013) were referred to by the researchers. Four faculty members from the mathematics education department and three mathematics teachers were asked to review the test for the content validity of the tests that were developed as well as the accuracy of the items. Necessary corrections were made based on the feedback received and the test was finalized. A holistic evaluation form was developed based on the TMoNE (2011) and the evaluation of the test at the end of the implementation was carried out using this form.

Open-ended questionnaire

After the model was implemented in the classroom, a questionnaire was administered to the students with the intention of examining its feasibility. Before the questionnaire was prepared, the relevant literature was reviewed, the views of two experts from mathematics education were considered and draft forms were developed. During the process of piloting, the questionnaire was distributed to 62 students. The questionnaire gathered at the end of the piloting was examined, necessary corrections were made and the questionnaire was finalized to be used in the research study. The addition of the following question to the questionnaire can be given as an example of the corrections that were made: "Do you have any other opinions and suggestions about the implementation process?" One of the questions included in the open-ended questionnaire was: "What effects did the DMS and the cooperative learning model have on you?"

Implementation process

A practice teacher participated in the workshop and he volunteered to implement the model in his class. A guide book which could be used as a supplementary resource about DMS was prepared by the researchers was distributed to the teachers. The workshop consisted of 18 sessions; each session lasted 50 minutes and was held on four days. The aim of the first 11 sessions of the workshop was to introduce the software. In the last seven sessions, information about the implementation of the cooperative learning model supported with DMS was given.

Figure 1. The implementation of the model in the learning environment

Every four students used one computer and two worksheets in order to enforce teammates to work together (Slavin, 1994). The researchers downloaded the materials which were planned to be used that week before the lesson. Base scores, determined based on students' final grades in the previous semester, were used for determining initial scores. Teams were composed based on these base scores and the chart developed by Slavin (1994). After the teams were identified, the volunteer teacher introduced the model in his class. How the model helped to teach difficult mathematical concepts more effectively, is presented in detail in Figure 1.

Worksheets and dynamic materials

Ten dynamic materials and six worksheets were developed about the topic on sequences. For quadratic functions, six dynamic materials and four worksheets were developed. While developing the dynamic materials and worksheets, the website http://www.geogebra.org/ and Hohenwarter and Hohenwarter (2012), TMoNE (2011), TMoNE (2013), Şişman, Lökçü, Oğuz and Atak (2013) were referred to. After the worksheets and dynamic materials were developed by the researchers, two mathematics educators and two mathematics teachers were asked to review them. Based on the feedback from the experts, the worksheets and dynamic materials were revised. Then, the materials were reviewed in the workshop with the teacher who volunteered to implement the model in his class. At the weekend meetings some corrections were made. Some examples from the worksheets which were used with the dynamic materials are given the Appendix.

Data analysis

Both qualitative and quantitative data were used together in the research. Quantitative data were obtained from the SKT and QFKT results. As the study group consisted of less than 50 people, the Shapiro Wilk test was used to check whether the quantitative data obtained from the measurement results came from a normally distributed population (Büyüköztürk, 2011). The results of the Shapiro–Wilk test were as follows: ($p_{pretest\text{-}Experimental\text{-}I} < .05$; $p_{pretest\text{-}Control\ Group} < .05$; $p_{posttest\text{-}Experimental\text{-}I} < .05$; $p_{posttest\text{-}Control\ Group} < .05$; $p_{pretest\text{-}Experimental\text{-}II} > .05$; $p_{posttest\text{-}Experimental\text{-}II} > .05$). In addition, the Q-Q, box and whisker plots, detrended normality plot, kurtosis and skewness values were analysed to determine whether the measurement results exhibited a normal distribution or not (Field, 2009). Measurement results of both groups have to exhibit a normal distribution to choose parametric tests (Büyüköztürk, 2011). Therefore, the Mann–Whitney test and dependent t-test were used for analyzing the quantitative data. The SPSS 18.0 program was used to for the analysis of the quantitative data obtained from the research. The value of 0.05 which is frequently used in education research studies is considered as the minimum significance level.

The qualitative data obtained from the open-ended questionnaires were analysed using content and descriptive analyses methods. The data obtained in the descriptive analysis were examined in detail using content analysis. Summarized data in the descriptive analyses were coded in order to examine the data in depth. Codes were used for different categories. The researchers consulted expert academicians to finalize the coding process and constitution categories (Yıldırım & Şimşek, 2011). Moreover, the number of students was taken as the frequency in the data analysis. "f" stood for the frequency of occurrence of the codes. Volunteer students for the sequences were codes as S1, S2, S3, …, S19. In addition, volunteer students for the quadratic functions topic were coded as Q1, Q2, Q3, …, Q25. The findings obtained as a result of the analysis of the data obtained from students included descriptions using these codes.

Findings

The Mann–Whitney test was used to examine whether there was a significant difference between the students in the experimental group and the control group based on their pre-test and post-test scores of the SKT. The students' pre-test and post-test results are presented in Table 1 and Table 2 respectively.

Table 1. Mann–Whitney test results of students' pre-test scores of SKT

Group	*n*	Mean rank	Sum of rank	U	*z*	*p*
Experimental-I	19	18.92	359.50	153.50	-.257	.797
Control	17	18.03	306.50			

According to Table 1, the Mann-Whitney test indicated that there was no significant difference between the experimental-groups$_S$ (*Mdn* = 14) and the control group (*Mdn* = 16) based on their pre-test scores, $U = 153.50$, $p > .05$, $r = -.04$. In terms of these results, both the experimental-groups$_S$ and the control group were equivalent based on their pre-test scores.

Table 2. Mann–Whitney test results of students' post-test scores of SKT

Group	*n*	Mean rank	Sum of rank	U	*z*	*p*
Experimental-I	19	22.26	423.00	90.00	-2.281	.023
Control	17	14.29	243.00			

According to Table 2, the Mann-Whitney test indicated that the students of the experimental-groups$_S$ (*Mdn* = 36) had significantly higher scores than the students of the control group (*Mdn* = 20), $U = 90.00$, $p < .05$, $r = -.38$). Based on these findings, it can be stated that the cooperative learning model supported with DMS is an effective model to increase student achievement in the sequences topic.

The dependent *t*-test was used to examine whether there was a significant difference between the students' scores in the pre-test and the post-test of the QFKT. The students' test results are presented in Table 3.

Table 3. Dependent *t*-test results of students' pre-test and post test scores of QFKT

Measurement	*n*	Mean	*SD*	*df*	*t*	*p*
Pre-test	24	17.62	11.27	23	-2.30	.030
Post-test	24	20.79	11.25			

The analysis results given in Table 3 reveal that there was a significant difference in the scores for the post-test (M = 20.79, SD = 11.25) compared to the pre-test (M = 17.62, SD = 11.27), $t(23) = -2.30$, $p < .05$, $r = .43$. These results suggest that the cooperative learning model supported with DMS is an effective model to increase student achievement in quadratic functions.

As a result of the analysis of data obtained from the open-ended questionnaire of the 43 students who volunteered in the study after the implementation of the model, a total of four categories were identified: "contributions of the model," "Obstacles to the implementation of the model," "Materials," and "Topics and lessons." Table 4 presents the codes and categories of these codes as well as the frequency (f) of students involved.

Table 4. The evaluation of the model based on the framework of the open-ended questionnaire

Category	Code	*f*
Contributions of the model	Enabling better understanding	15
	Visualizing the course	14
	Increasing retention	13
	Concretizing the course	8
	Pleasant and enjoyable learning environment	7
	Promoting interest and motivation	7
	Facilitating learning	6
	Having efficient lessons	5
	Learning from team work	4
	Learning the concepts via discussion	4
Obstacles to the implementation of the model	Taking time to get used to it	6
	Lack of computer skills	3
	Disagreements within the group	1
Materials	Suitability of materials	34
Topics and lessons	Implementing it in other courses	4
	Implementing it all the concepts of mathematics	3

When Table 4 is examined, for the framework of "contributions of the model," students' positive views regarding the cooperative learning model supported with DMS stand out. Students stated that as a result of the model, they understood the lesson better, the model visualized the course, it created a pleasant and enjoyable learning environment, and it increased retention. Moreover, they stated that the model did not have any difficulties, while their interest and motivation increased with the use of the model, and the lessons became more efficient.

A total of 15 of the students stated that the model promoted better understanding. Some students' views were as follows:
"… We refreshed our knowledge. At the same time, it enabled us to understand the subjects better …." (S1)
"… I understood a topic better for the first time."(S2)
"I could not understand the topic that I want to learn in the classroom. But using the cooperative learning model and DMS together provided better understanding of difficult mathematics concepts." (S8)

Fourteen of the students stated that the model enabled them to visualize the course. Two students' views exemplifying this situation are:
"One of its advantages for the mathematics course is that it can be visualized with models …"(S4)
"Its implementation in the mathematics course was good both for me and my friends because the mathematics course became more visual and concrete." (S10)
Eight of the students stated that the model concretized the course. Two students' views exemplifying this situation are:
"Since the model provides concretization our knowledge is more permanent" (Q6)

"In my opinion it is very effective, we see abstract mathematics concepts visually. We concretized the mathematics concepts in our team…" (Q14)

Thirteen students stated that the model increased retention and they did not have any difficulties. The following are some students' views:
"We understood much better. We both loved mathematics and we learned that it was much simpler. Moreover, thanks to the cooperative learning model and dynamic mathematics, we remembered easily. Therefore, therefore the two combinations provided a positive learning environment for the sequences topic" (S 17)
"The use of both GeoGebra and cooperative learning approaches is a very effective way to increase retention." (Q2)
"It is more permanent because of hands-on learning" (Q24)
"I think that this study did not pose any difficulties to us because this study made it much easier for us." (S 10)

Seven students mentioned that the model created a pleasant and an enjoyable learning environment. Two students' views are as follows:
"I believe that it has positive effects on me and my peers. It is a kind of enjoyable learning technique in which we listen and implement without getting bored and it really has a lot of benefits." (S5)
"I realized that mathematics was pleasant, fun, and enjoyable." (S11)

Two of the seven students who stated that the model increased their interest and motivation shared their views as follows:
"The combination of dynamic mathematics software and cooperative learning increased my interest and motivation, and this combination provided better understanding of quadratic functions." (Q1)
"In fact, working in groups was very beneficial. As I refreshed my knowledge, my question repertoire expanded and my interest in mathematics increased." (S3)

When Table 4 is examined regarding "obstacles to the implementation of the model", it is found that the students had difficulties with the cooperative learning model supported with DMS and these difficulties usually resulted from disadvantages such as lack of computer skills, taking time to get used to the model, and disagreements within the group. Two students' views can be given as examples of this category:
"At the beginning I did not understand anything but sometime later I began to understand." (S2)
"At first we had difficulty in using the materials, but later we got used to them… Because they were easily understood, we liked them a lot." (S5)

When Table 4 is examined regarding the "materials" category, most of the students stated that in the cooperative learning model supported with DMS, the materials used with the model were suitable. Two students' views on this issue are given below:
"Dynamic materials about sequences brought different perspectives to the topic in order to learn that subject…" (S12)
"According to me, we learn better quadratic functions thanks to dynamic materials." (Q4)

When Table 4 is examined regarding "topics and lessons", students stated that they did not want the cooperative learning model supported with DMS to be implemented with only mathematics topics but with all other courses as well. The following view of S15 is an example:
"I wish we had such implementation with all courses. As we particularly have difficulties in numerical courses and sometimes we are afraid of our teachers, it would be much more productive if we worked with our peers."

The feasibility of the cooperative learning model supported with DMS in classes was examined based on students' views. It was revealed that the students understood the course much better with the model and it created a pleasant and an enjoyable learning environment. Moreover, the students suggested that the model increased retention and it affected their interest and motivation positively. In addition to this, they stated that the model did not have any difficulties, and it was found that the materials used with the model were instructive and they could be used in the classroom. In this context, when the views of Q1, Q2, S8, and S17 were examined, they stressed that the combination of the cooperative learning model and the dynamic mathematics software provided these positive contributions. However, the students mentioned that there could be difficulties with the implementation of the model because of some reasons such as disagreements within the group, taking time for them to get used to the model, and lack of computer skills.

Results and recommendations

In this study, the effect of the cooperative learning model supported with DMS on student achievement of difficult mathematical concepts such as sequences and quadratic functions was investigated. In addition, student views about this model were examined in detail. The analysis of quantitative data showed that this integrated model increases student achievement in the sequences and quadratic functions topics. This result is similar with previous researches that examined the use of cooperative learning in mathematics classrooms (e.g., Bernero, 2000; Kumar & Harizuka, 1998; Nichols & Miller, 1994; Stevens & Slavin, 1995; Ural, 2007; Vaughan, 2002). However, these studies generally focused at the elementary mathematics level. Similarly, this result is parallel with the previous researches which investigated the use of DMS in mathematics classrooms (Tatar & Zengin, 2016; Thambi & Eu, 2013; Zengin & Tatar, 2014; Zengin & Tatar, 2015). But these studies do not explain how to use DMS effectively in a cooperative learning environment that plays a very significant role in acquiring one of the important skills in the 21st century. Considering the quantitative results about sequences and quadratic functions, the cooperative learning model supported with DMS played significant role in increasing student achievement in these difficult mathematical topics. This model may contribute to students' learning of difficult mathematical concepts from elementary level to university level.

It was found in other studies that students understood concepts better in the settings where DMS was used (Arbain & Shukor, 2015; Thambi & Eu, 2013; Zengin & Tatar, 2014; Zengin & Tatar, 2015) and their interest and motivation in the course increased (Choi, 2010; Green & Robinson, 2009; Zengin & Tatar, 2014). DMS help students achieve better understanding and conceptual learning (Tatar & Zengin, 2016). Similarly, it was revealed by other researchers that cooperative learning increased the motivation levels of students (Slavin, 1987; Nichols & Miller, 1994). It can be implied that because the model uses software and cooperative learning together, the results of this study are parallel to the results of other studies that have been conducted. It was found based on students' views that the visualizations used in the model made the concepts clearer and an enjoyable learning environment was created with the model. The finding that the lessons which used DMS became more visual (Thambi & Eu, 2013; Diković, 2009) also coincide with the results of this study due to the software included in the model.

The model can enable students to understand concepts better and more easily by visualizing the concepts, creating a pleasant and an enjoyable learning environment, and increasing student motivation and interest and thus affecting students' performance positively. It can be inferred that the cooperative learning model supported with DMS provides a good alternative learning model for understanding difficult mathematics concepts. Thus, teachers may have beneficial opportunities to use educational technology in mathematics classroom.

Based on the qualitative data obtained, students' lack of computer skills must be addressed so that the model can be more applicable. It is important that students must be introduced to basic ICTs in computer courses and also they must be taught the content of the basic software specific to the field and how to use them in education. Accordingly, the content of computer courses can be updated. Moreover, it is observed that it will take some time for students to get used to the model and they can have disagreements within the group. It is suggested that teachers should implement the model and design the materials and worksheets considering these disadvantages. For this purpose, it is recommended that teachers should use relevant internet resources and also visit the official website of GeoGebra (www.geogebra.org) so that they can design the dynamic materials and worksheets themselves.

If researchers who would like to carry out studies on the model focus on how to eliminate the problem of time, disagreements within the group and lack of computer skills, they can make positive contributions to the model. Moreover, researchers can make contributions to the field by examining the effect of the model on students and teachers in other subjects.

Limitations

The main limitation of this study is about the quantitative part of the study. The quantitative part of study regarding quadratic functions was designed as a single group pretest-posttest. Additionally, in this study, the cooperative learning model and DMS were used together in the learning environment. When the codes of the findings of the qualitative data were investigated it was seen that DMS played a significant role. The reason why DMS was highlighted in the findings was so that the self-reported data obtained may enable students to focus only on this software.

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Appendix 1

Worksheet –Example about sequences

Group name: *Group members:* *Date :…………………..* *1)………………………………………………………* *2)……………………………………………………….* *3)……………………………………………………….* *4)……………………………………………………….* *You can ask for advice from your teacher about the problems you have encountered in the worksheet while using the dynamic materials. Carry out your work with your group members. Please do not hesitate to write your opinions.*

Let's look at the M1 material.

Write the number of points in the table when the slider takes the values 1, 2, 3, …, 7

n values	Number of points (a_n)	What is the relationship between the n values and number of points?
1		
2		
3		
4		
5		
6		
7		

Let $f(1)=a_1$, $f(2)=a_2$, $f(3)=a_3$, … and $f(n)=a_n$. Write the values of function f as $(a_n)= (a_1, a_2, a_3, \dots , a_n)$.
Let $f(n)=a_n$ Can we show any function $f:N^+ \rightarrow R$ as $(a_n)= (a_1, a_2, a_3, \dots , a_n)$? Explain.

Let's look at M2 material.

You can get different visions by varying the sliders a and b in the graph-1 and graph-2 windows. What is the difference between the graph-1 and graph-2 windows? Explain.

Find whether the following functions $N^+ \rightarrow R$ are sequences or not.

a-) $f(n)=2n+1$

b-) $f(n)=\frac{n}{n+1}$

c-) $f(n)=\frac{n}{n-2}$

Which term of the sequence $(a_n) = \left(\frac{n-1}{n}\right)$ is equal to $\frac{5}{6}$?

Explain the difference between functions and sequences using graphs.

Screenshots of the materials M1 and M2, respectively that are used in worksheet-1 are given below.

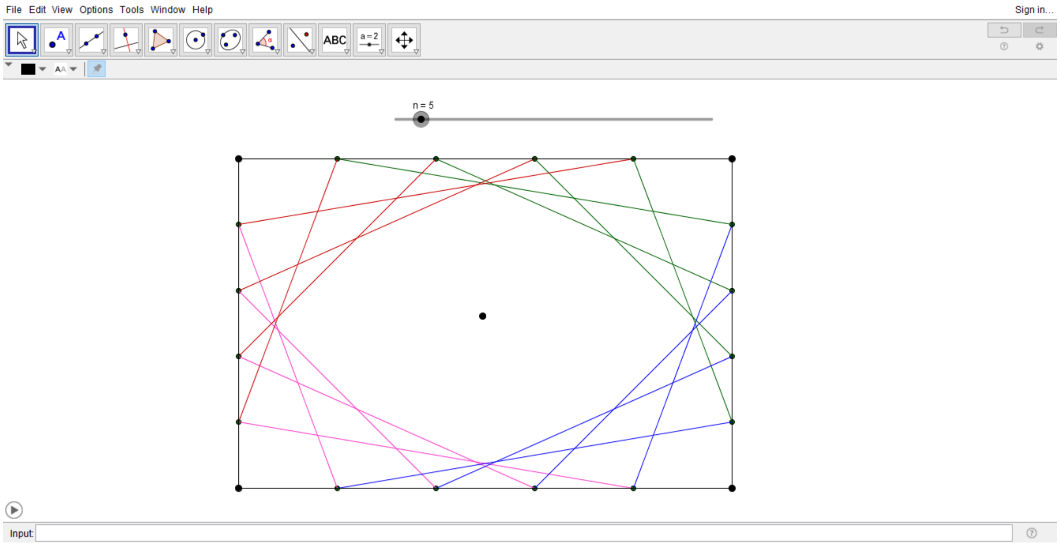


Figure 2. A screenshot of the material-M1

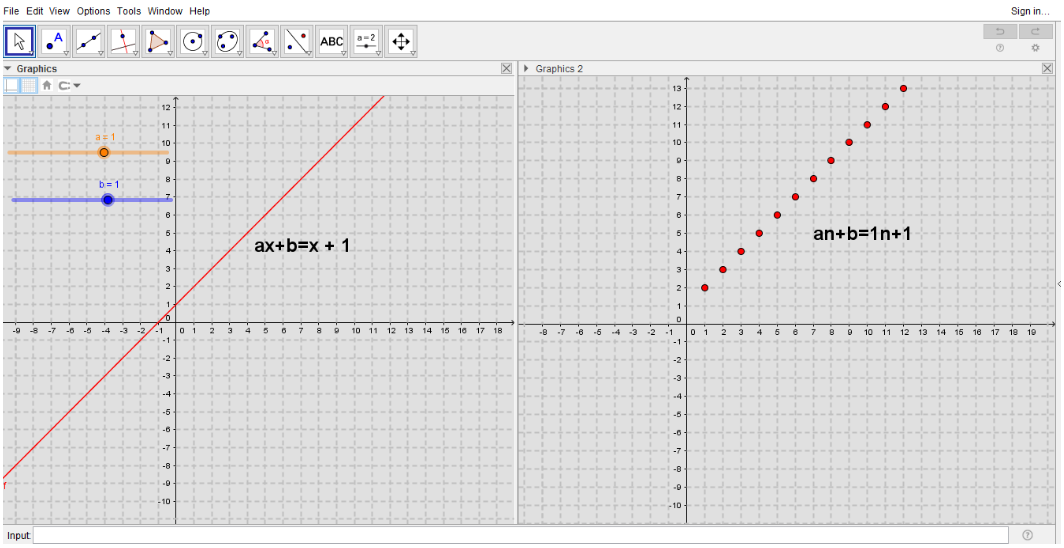


Figure 3. A screenshot of the material-M2

Appendix 2

Worksheet –Example about quadratic functions

Group name: *Group members:* *Date :…………………..* *1)………………………………………………………* *2)……………………………………………………….* *3)……………………………………………………….* *4)……………………………………………………….* *You can ask for advice from your teacher about the problems you have encountered in the worksheet while using the dynamic materials. Carry out your work with your group members. Please do not hesitate to write your opinions.*

Let's look at the M3 material

Use slider *a* in order to examine the graph of f ($f(x) = ax^2 + bx + c$). How does the slider impact the graph and the functions? Explain what are the relationships between the functions leading coefficient (*a*) and the graphs of functions?

Use slider *b* in order to examine the graph of f ($f(x) = ax^2 + bx + c$). How does the slider impact the graph and the functions? Explain what are the relationships between the values of *b* and the graphs of functions?

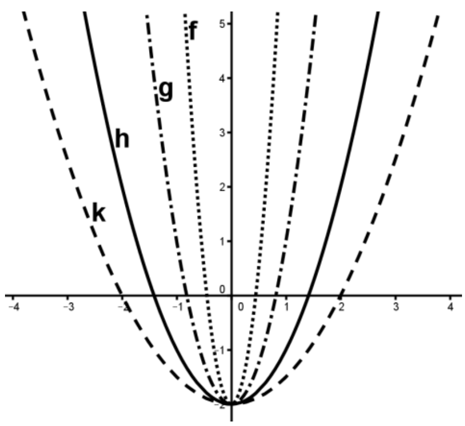
Use slider *c* in order to examine the graph of f ($f(x) = ax^2 + bx + c$). How does the slider impact the graph and the functions? Explain what are the relationships between the values of *c* and the graphs of the functions?

Let's look at the M4 material

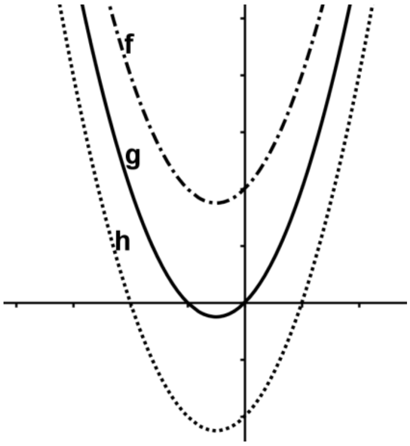
Use slider *a* in order to examine changes that occur in the graph of f ($f(x) = a(x-r)^2 + k$) in M4 material. Explain what are the relationships between the values of *a* and the graphs of functions?

Compare the peaks of various function graphs that are created from changing the leading coefficient (the values of *a*) of function ($f(x) = a(x-r)^2 + k$).

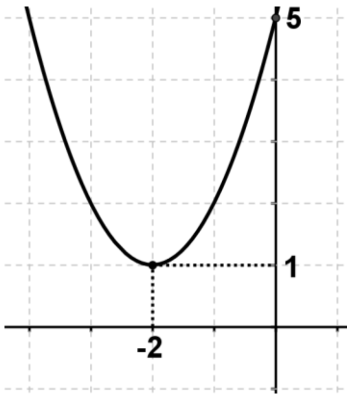
Ex: The functions *f, g, h* and *k* are in the form $ax^2 + bx + c$ and in the each four graphics only "*a*" can take different values. Compare the values of a_f, a_g, a_h, and a_k.



Ex: The functions *f*, *g*, and *h* are in the form ax^2+bx+c and in the each three graphics only "*c*" can take different values. Based on this compare the values of c_f, c_g, and c_h.



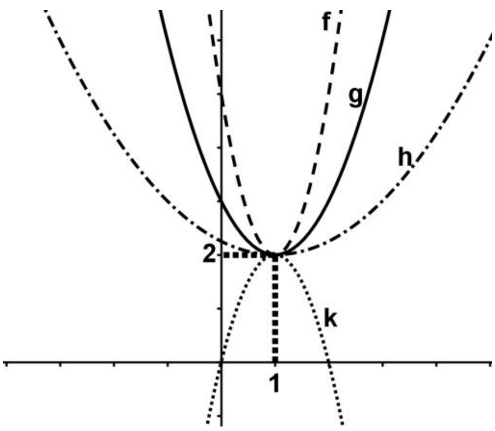
Ex: The graph of the parabola $y=a(x+2)^2+1$ is given in below. Find the value of "*a*" in the graph.



Ex: The parabolas *f*, *g*, *h*, and *k* are in the form $(f(x)=a(x-r)^2+k)$ and in the each four graphics only "*a*" takes different values. Then:

a) Sort the descending the values of a_f, a_g, a_h, and a_k.

b) Compare peaks of the parabolas *f*, *g*, *h*, and *k*.



Screenshots of the materials M3 and M4, respectively that are used in worksheet-2 are given below.

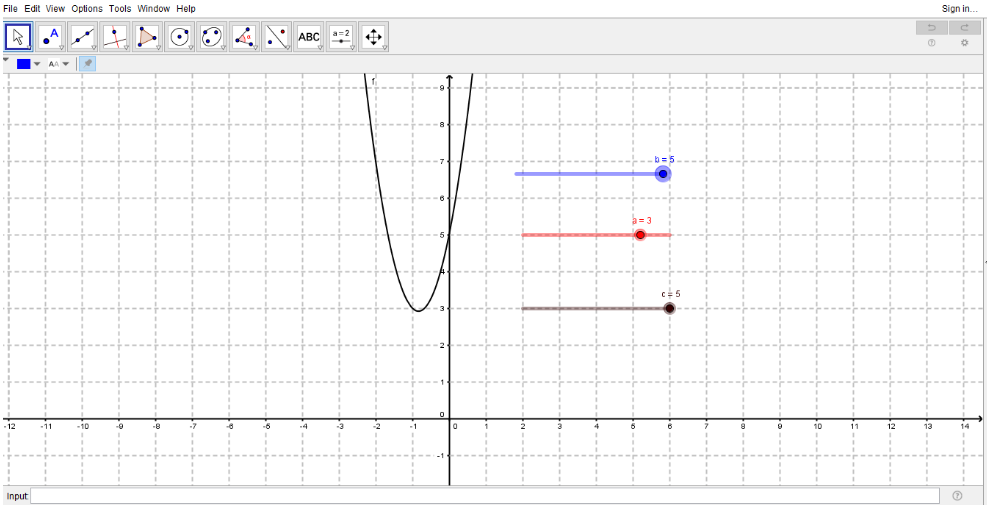


Figure 4. A screenshot of the material-M3

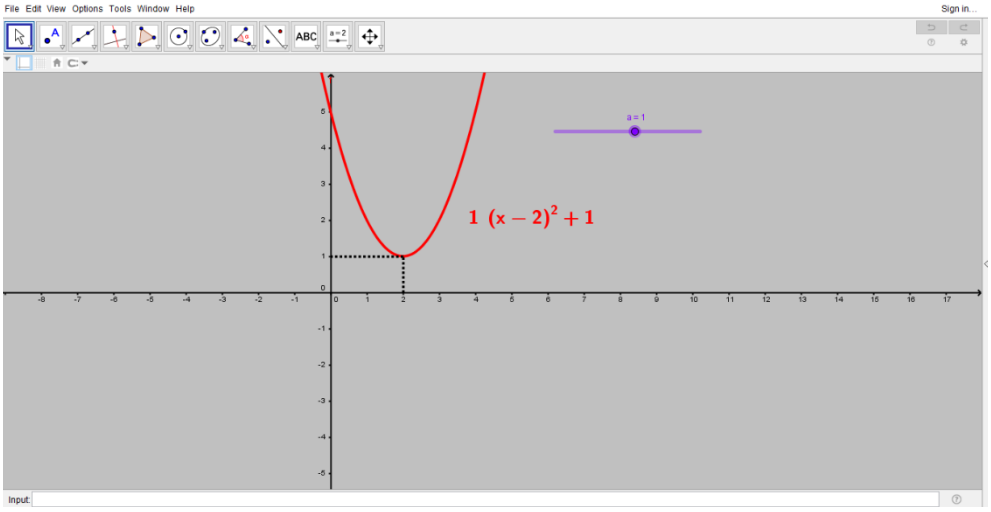


Figure 5. A screenshot of the material-M4