

Investigating the Use of the Khan Academy and Mathematics Software with a Flipped Classroom Approach in Mathematics Teaching

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ABSTRACT

The purpose of this study was to determine the effect of the flipped classroom approach designed by using Khan Academy and free open source software on students' academic achievement and to examine students' views about this approach. The research was evaluated in the light of both qualitative and quantitative data. Twenty-eight students studying in the department of mathematics education in a state university in Turkey comprised the study group of the research which was conducted using a mixed methods research design. A double integral achievement test and an open-ended questionnaire about the flipped classroom approach were used as data collection tools. A Wilcoxon signed-rank test was used for the analysis of quantitative data and content analysis was used to analyse the qualitative data. According to the analysis of the research it was found that the flipped classroom approach designed with using both the Khan Academy and mathematics software increased student achievement in double integral. It was also found that this learning approach enhanced students' understanding and provided visualization in mathematics teaching. Moreover, it was revealed that this approach promoted retention and made understanding much easier.

Keywords

Flipped classroom approach, Khan Academy, GeoGebra, Maxima, Double integral

Introduction

Information, communication and working styles have changed in the 21st century. This change has affected education and it has required computer and electronic technologies to be used in every field (Niess, 2005). Educators, particularly involving courses in mathematics that are difficult to understand (Freudenthal, 1983), have enabled students to better understand the concepts involved using technologies (Hoyles & Jones, 1998). The technologies have also given students opportunities to work on real life problems (Pierce & Stacey, 2011), and has also enabled them to identify different representations of concepts (Heid & Edwards, 2001).

It is regarded that using information communications technology (ICT) in very difficult mathematics courses is beneficial to students (Jones, 2000; Laborde, 1993; Marshall, Buteau, Jarvis, & Lavicza, 2012). Teachers use their knowledge of content, teaching and learning, and technology to promote experiences that develop students' learning and creativity in computer-mediated environments (International Society for Technology in Education [ISTE], 2008). Moreover, teachers are expected to prepare their content by using a variety of software and to transfer them to the learning environment with the use of worksheets. In addition to teachers' efforts in using these education technologies, it is important to consider how and with which approaches these technologies could be reflected in the classroom learning environment. It is considered that the flipped classroom, one of the blended learning models used widely, (Sahin, Cavlazoglu, & Zeytuncu, 2015) can enable teachers and students to structure the learning environment.

Essentially in a flipped classroom what is learned in class is learned at home, and homework done at home is now done in class (Bergmann & Sams, 2012). The traditional model of instruction is teacher-centred; the teacher gives lectures during the lesson and assigns students homework to do at home. The flipped classroom, or inverted classroom, reverses traditional education: the teacher delivers the content outside the classroom with videos prepared by him/her, and uses class time for active learning by having students collaborate and interact with each other (Mok, 2014). As a result of the flipped classroom, students find more opportunities to get engaged with more activities in class and to have discussions about the concepts involved. However, the teacher should very carefully plan activities, videos, presentations, or study notes to deliver content outside of the classroom.

There is also a concern that the flipped classroom can be regarded as one of the barriers between technology and teachers. However, Bergmann and Sams (2012) stress that the solution to overcoming the barriers in flipped classrooms is to employ, train, and support teachers. Moreover, although some critics fear that the Khan Academy's importance can result in standardization and deprofessionalization, Bergmann and Sams (2012) and Andrea Smith point out that educational videos as important tools because teachers can develop content, share

resources, and promote practice (as cited in Tucker, 2012). The Khan Academy provides numerous activities, instructional videos, and a personalized learning dashboard that enable students to study at their own pace in and outside of the classroom. The Khan Academy guides students from nursery class to advanced mathematics by using the most developed and adaptive technologies. Moreover, the educator dashboard offers a summary of class performance. The Khan Academy founded by Salman Khan has grown into an 80-person organization that aims at providing a free world-class education for anyone, anywhere (Khan Academy, 2016). The Khan Academy is translated into different languages and offers content suitable to all levels in an entertaining environment by taking into consideration students' knowledge gaps (Dijksman & Khan, 2011). It may be a suitable platform for students' studies outside of the classroom in the flipped classroom approach. Although the duration of the videos in the Khan Academy is not very long, usually lasting between seven and 14 minutes, they provide opportunities for students to identify, explain, and practice different mathematics concepts using relevant software (Thompson, 2011).

In the flipped classroom approach, students benefit from the rich content of the Khan Academy at home before the actual class session, but it becomes more important that the kind of learning environment they will encounter in the classroom under the guidance of the teacher will enable them to enhance their understanding of mathematical concepts. If the preparations done by students outside of the classroom are not complemented with planned activities and with the teachers' guidance in class, the positive effects of the flipped classroom may not emerge as expected. Thus, even if students understand the general framework of the concepts outside of the class, in order to examine the different representations and different situations of the concepts, the teacher is required to come to the class prepared. When the courses in the Khan Academy are examined, it is seen that some contents related to mathematics are enriched by using different software. Thus, students must be offered contents with relevant software. They must also be given opportunities to have different experiences with the software and to reinforce the concepts they have learned outside the class by means of discussions with their peers under the guidance of the teacher in the classroom environment. There are numerous open source software programs which promote the design of such learning environments in mathematics teaching. Maxima (Gaertne, 2005), GeoGebra (Hohenwarter & Fuchs, 2004), SageMath (Stein, 2006), Cadabra (Peeters, 2007), GNU Octave (Eaton, 1997) and Axiom (Pfaffman, 2007) are examples of such systems. Open source programs offer alternative contents to students and teachers for designing the learning environment.

The flipped classroom can provide higher order exploration opportunities for students if the activities in which students engage in the class are well-planned under the guidance of the teacher. Thus, benefiting from mathematics software with in-class activities can promote the efficiency of the flipped classroom strategy. In this study, Maxima and GeoGebra were actively used during the in-class implementations of the flipped classroom. Maxima and GeoGebra were preferred as open source mathematics software because they offer a variety of languages and are easy to use and set up. It is considered that students' experiences about the flipped classroom approach can contribute to mathematics research and teaching. Few studies have been carried out with the flipped classroom approach in mathematics education (Love, Hodge, Grandgenett, & Swift, 2014; Moore, Gillett, & Steele, 2014; Sahin, Cavlazoglu, & Zeytuncu, 2015). These studies have provided limited explanations on how in-class and out of class activities of the flipped classroom approach were designed. Moreover, these studies resulted from out of school settings like in the Khan Academy and YouTube; however, using mathematics software which has a positive effect on learning in the classroom effectively has been mentioned in very few studies. In this study, the students who used the Khan Academy videos in out of class settings learned mathematics in informal cooperative activities by focusing on the double integral subject which they had difficulties in understanding. The flipped classroom approach supported with the efficient use of the Khan Academy materials in out of class settings and with mathematics software in in-class learning is important as it enables students to benefit from technology effectively while learning concepts with higher levels of difficulties. Moreover, the flipped classroom approach in mathematics education is usually used in middle and high school levels (Chen, Yang, & Hsiao, 2015; Fulton, 2012; Heo & Choi, 2014) but there is lack of research at the university level. The studies at university level are usually carried out involving students' perceptions and attitudes and instructors' experiences about the flipped classroom approach (Cilli-Turner, 2015; Love, Hodge, Grandgenett, & Swift, 2014; Ogden, 2015; Palmer, 2015). Due to the limited number of studies that have been conducted on students' achievement, Ogden (2015) highlights that the flipped classroom approach needs to be examined in greater depth and its effects on students' achievement should be investigated. Therefore, the effects of using the Khan Academy materials within the flipped classroom approach, by using a free and an open source mathematics software, were examined. In particular, the study involved the topic of the double integral, one of the topics which students have difficulties with and is considered to be one of the most important topics of calculus (Mathews, 1990). In this context, the purpose of the study was to determine the effect of the flipped classroom approach designed using the Khan Academy and mathematics software together on students'

achievement and to examine students' views about this approach. With this aim, the research questions of the study are as follows:

- Research question 1 (RQ1): What is the effect of the flipped classroom approach designed by using the Khan Academy materials and mathematics software on student achievement in the double integral topic?
- Research question 2 (RQ2): What are students' views on the use of the Khan Academy materials and mathematics software with the flipped classroom approach in mathematics courses?

Method

The study was carried out using a mixed methods design involving both quantitative and qualitative data. The design aimed at gathering both qualitative and quantitative data simultaneously or sequentially. The reason for gathering qualitative data is to supplement the primary quantitative data. Most examples in the literature reveal that qualitative data are embedded within the quantitative data (Creswell, 2012). In this study, the effect of the flipped classroom approach designed using the Khan Academy materials and mathematics software on students' achievement was revealed by the quantitative data. The quantitative part of the research study was designed with a single-group using a pre-test and a post-test. Before the flipped classroom approach was implemented, a pre-test was administered. A post-test was then administered after implementation of the flipped classroom. Both the pre-test and post-test were the same tests (McMillan & Schumacher, 2010). Qualitative data were used to explain the students' views on the flipped classroom approach.

Study group

Twenty-eight students studying in the department of mathematics education in a state university in Turkey comprised the study group of the research. The student consisted of 10 males and 18 females. Participants' ages ranged from 22 to 26 years. The convenience sampling method was used in selecting the study group. In convenience sampling, subjects are selected because of their accessibility or convenience. Convenience samples are commonly used in both quantitative and qualitative studies; many researchers prefer this sampling technique because of its easy accessibility, efficiency, and is free from practical constraints (McMillan & Schumacher, 2010).

Data collection tools

The Double Integral Achievement Test (DIAT) and an open-ended questionnaire about the flipped classroom approach were used as data collection tools. The DIAT was developed taking into account the opinions of two mathematics educators and two mathematics experts. Moreover, the questions in the DIAT were taken from Thomas, Weir and Hass (2011), Balcı (2009) and Balcı (2010). Some of the questions in the DIAT are as follows:

- Is it possible to calculate the integral of a continuous function $f(x,y)$ on a rectangular domain in the xy plane and obtain different results depending on the order of integration? Explain your answer with your reasons.

For the following integral;

$$\int_0^2 \int_{x^2}^{2x} (2x+1) dy dx$$

- Draw the integration domain;
- Write an equivalent integral by changing the order of integration.

The DIAT was evaluated using a holistic rubric. This rubric was developed by the researcher with help from the Turkish Ministry of National Education [TMoNe], (2013). The maximum score that could be obtained in this achievement test was 30. The qualitative data in the research were collected using an open-ended questionnaire. When the open-ended questionnaire was developed, two mathematics educators' opinions were taken into account and a pilot test was carried out. After piloting, this questionnaire was finalized. One of the questions included in the open-ended questionnaire was: "What are the differences between your views before and after the implementation of the flipped classroom approach in the study of double integrals? Please explain."

Implementation

The research study lasted eight weeks. In the first five weeks, students learned how to use GeoGebra and Maxima at a basic level. In the remaining three weeks, the topic of double integrals was taught using the flipped

classroom approach. In the first five weeks, implementations were carried out in 10 course hours (10 x 50 min.), with 2 hours in class per week and in the last three weeks. Instruction lasted for a total of nine course hours (9 x 50min.), with three hours per week. The implementations were carried out in a total of 19 course hours (19 x 50min.). Students watched the double integral videos from the Khan Academy outside of the class for three weeks before coming to class. Students studied in the computer lab using the worksheets prepared by the researcher in each lesson. Lessons in the computer lab are shown in Figure 1.

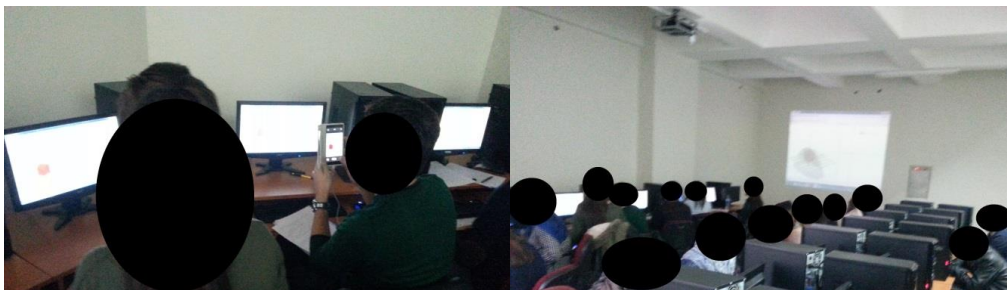


Figure 1. Pictures from the class environment

The researcher played the role of a guide when students used the mathematics software and worksheets. The researcher prepared three worksheets and four dynamic materials. The materials developed using mathematics software or by the students in the class were used with the worksheets. The worksheets were designed to provide opportunities for students to build and use materials. While developing the worksheets and materials, the websites <http://tube.geogebra.org/> and <http://maxima.sourceforge.net/> and Balcı (2009), Balcı (2010) were referred to. Hohenwarter and Hohenwarter (2012), Ku (2015), Thomas, Weir and Hass (2011) were also referred to. After the worksheets and materials were developed by the researcher, two mathematics educators who were experts in this field were asked to review the materials and the materials were revised based on feedback provided. Appendix 1 consists of some examples from the worksheets and related materials. A sample of the materials used in the lesson is given in Figure 2.

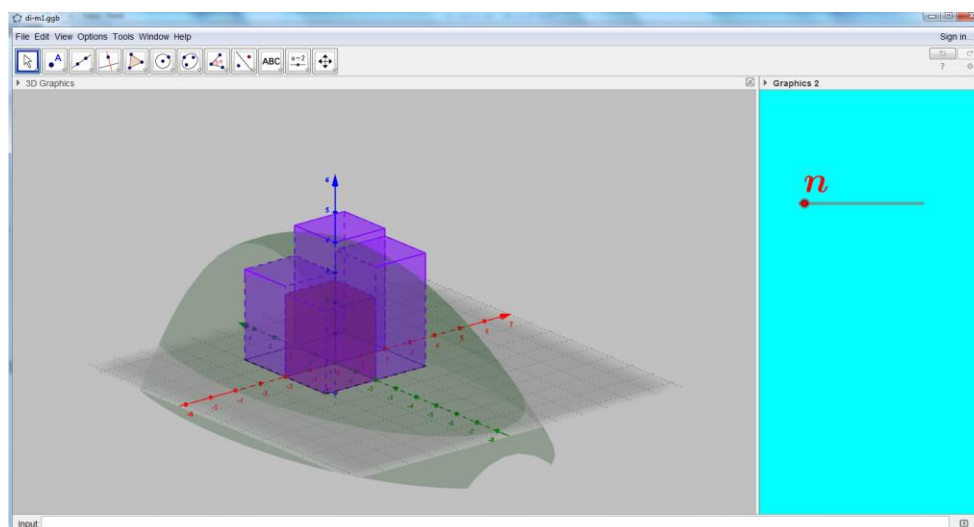


Figure 2. A Dynamic material about approximating solids with rectangular boxes

Here, using a slider, students were asked to change the n values. As the n value increased, they were asked to explain the relation between the volume of the solid obtained with the Riemann sum. An example of a material obtained with the increase of n -value is given in Figure 3.

During the lesson taught using the flipped classroom approach, informal cooperative activities were conducted in stages when students used mathematics software using the worksheets. Spontaneous group discussions and the think-pair-share technique were used in the informal discussions. The researcher asked students at various times during the lesson what mathematics concepts mean, why something works and how a mathematics problem may be solved in a spontaneous group discussion. This group discussion was used during the first week. In the second and third weeks, the researcher used the think-pair-share strategy. The students paired with another student within their teams. The researcher posed questions using the mathematics software and worksheets to the class. The students were required to think of an answer on their own, then to pair with their partners to concur on a

solution. In the end, the researcher asked students to share their answers or thoughts with the class (Slavin, 1995). Thus, students were expected to come to the lesson prepared with Khan Academy materials and under the guidance of the researcher students had more opportunity to focus on and discuss the topic of double integrals.

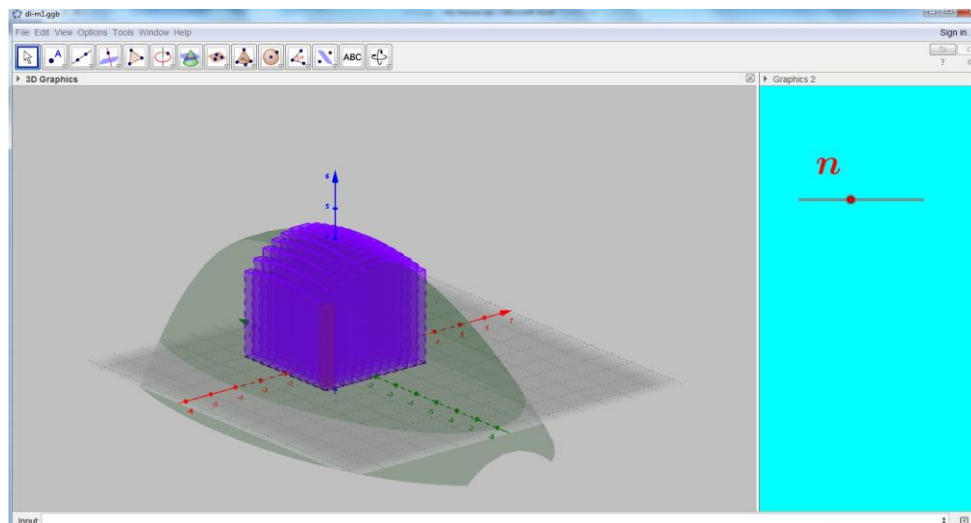


Figure 3. With the increase of n-value, the Riemann sum approximations approach the total volume of the solid

Data analysis

The data of the research were obtained from the DIAT. Normality analysis of the quantitative data obtained from the DIAT was performed. The Shapiro–Wilk test was performed and Q-Q plot, box and whisker plot, kurtosis and skewness coefficients were examined (Field, 2009). The results of the Shapiro–Wilk test were as follows ($p_{pretest-DIAT} < .05$; $p_{posttest-DIAT} < .05$). Analyses of the test results, as well as the Q-Q plot, box and whisker plot, kurtosis and skewness coefficients revealed that both the pre-test and the post-test did not show normal distribution. For this reason, the Wilcoxon signed-rank test was used to examine the differences between the pre-test and the post-test. The SPSS 18.0 program was used for the data analysis. Regarding the significance level, the value of .05, which is most frequently used in educational studies, was considered. For the calculation of the effect size, the $r = z / \sqrt{N}$ equation was used (Field, 2009), while the following criteria for the values of r [$r = .10$ (small effect), $r = .30$ (medium effect) and $r = .50$ (large effect)] were considered for the effect size (Cohen, 1992).

Content analysis was used for the analysis of the qualitative data. Categories were created by coding student responses to each question in the open-ended questionnaire. These were presented in tables with the frequencies and percentages. Moreover, regarding the categories created, the sample quotations taken from the students were coded as S1, S2, S3 ..., S28.

Findings

In the research, the DIAT was used as a pre-test before the experimental treatment and as a post-test after the experimental treatment to determine the effect of the flipped classroom approach designed with using Khan Academy materials and mathematics software for the implementation of the topic of double integrals on students' academic achievement. The Wilcoxon signed-rank test was used to examine whether or not there was a significant difference between the scores students achieved in the pre-test and post-test. Students' test results are given in Table 1.

The analysis results presented in Table 1 demonstrate that the scores students achieved from the double integral achievement test after the implementation of the flipped classroom approach ($Mdn = 23$) were at a significantly higher level than before the implementation ($Mdn = 3$) scores of the test, $z = -4.21$, $p < .05$, $r = -.62$. Moreover, when the mean scores of the pre-test and post-test were examined, students' pre-test mean score was 1.69. After the intervention students' post-test mean score was 21.82. Based on these results, it can be stated that the flipped classroom approach designed using the Khan Academy materials and the mathematics software was an effective approach to increase students' achievement.

Table 1. Wilcoxon-signed rank test results of the students' pre-test and post-test scores

Post-test pre-test	<i>N</i>	Mean rank	Sum of ranks	<i>z</i>	<i>p</i>
Negative rank	0	.00	.00	-4.212*	.00
Positive rank	23	12.00	276.00		
Ties	0	-	-		

Note. *Based on negative ranks.

As a result of implementation of the flipped classroom approach, three categories called “advantages,” “students’ suggestions,” and “disadvantages” were created from the analysis of the data obtained from the open-ended questionnaire responded to by the 28 volunteer students. Table 2 presents the codes and the categories belonging to these codes as well as the frequencies (*f*) of students involved.

Table 2. Evaluation of the approach

Category	Code	<i>f</i>	%
Advantages	Enhancing understanding	22	78.57
	Visualization	17	60.71
	Promoting retention	17	60.71
	Making understanding much easier	15	53.57
	Coming to the class prepared	13	46.42
	Concretization	8	28.57
	Making difficult concepts easy	8	28.57
	Liking the topic	7	25.00
	Moving away from memorization	6	21.42
	Understanding faster	6	21.42
	Promoting productive learning	4	14.28
	Increasing motivation	4	14.28
Students’ suggestions	Using Khan Academy and mathematics software together	23	82.14
	Must be implemented in all subjects	7	25.00
Disadvantages	The inadequacy of using computers	5	17.85
	Difficulties in using mathematics software	2	7.14

When Table 2 is analysed in terms of the “advantages,” it emerges that the flipped classroom approach designed using the Khan Academy materials and mathematics software together enabled students to understand the concepts better. Similarly, it was emphasized by the students that the approach promoted visualization and retention of the knowledge learned and it fostered learning of concepts more easily. In addition to these, Table 2 suggests that the approach helped students to come to the lesson prepared, it concretized the subject and also students loved it while at the same time it moved students away from memorization.

Most of the students stated that the flipped classroom approach enabled them to understand the subject better. The views of S7 can be given as an example:

“Regarding the preparedness level, we come to the class in a much better way and we watch the subjects which we are going to study via computer on videos and then we study them in the class. This has a positive effect on my learning. I noticed that I comprehended much better.”

The reason why students understood the concepts much better with the flipped classroom approach designed using the Khan Academy materials and mathematics software is that this approach provides more visuals in mathematics courses and students are prepared before coming to the class. The views of S2 and S25 supporting this opinion can be given as examples:

“When subjects are supported with visuals, they became more understandable. I could construct the examples much more easily with my logic...” (S2)

“I believe that this model has been very helpful for us to come to the lessons prepared. Because it enhances the level of our prior knowledge, it enables us to learn more clearly and permanently...” (S25)

A total of 60% of the students stated that the approach promoted visualization and retention of knowledge. The following views of two students can be given as examples.

“As it is supported with visuals, it is remembered easily and it moves us away from memorization.”(S2)

“As we see visuals, they are remembered easily and I do not forget what I see. I am so lucky about this issue. It enables me to remember easily. I understood double integral very well and I will never forget it.” (S10)

When the views of S2 and S10 are examined, it is revealed that in this approach, the visuals provided before the lesson and during the lesson promoted retention of the concepts learned. Two students' views supporting this can be given as follows:

"It was proved once again that the visuals enhanced retention. I could not solve integral before but now I can solve double integral.... Long live flipped mathematics classroom." (S9)

"Examining mathematics and viewing it in detail (with the support of 3D) on computer environment increase." (S11)

As many as 53% of the students stated that they understood the concepts much more easily with this approach. The following views of two students can be given as examples:

"Before the implementation, I was so scared. After the implementation, I understood that the subject was easy...I will do it with inner peace." (S14)

"Thanks to this model, we understood that those subjects which are difficult to comprehend like double integral would be apprehended more easily." (S22)

Only 46% of the students stated that they came to the class prepared because of the approach. S28 shared his opinion as follows:

"This model enables students to be active in the learning process. Thus, more permanent learning is actualized. In addition, coming to the class prepared for the subject which we will learn in the class increases more interest in the course. It promotes motivation."

Considering the views of S28, it may be concluded that students were of the opinion that their motivation towards the course will increase when they come prepared to the lesson.

Only 28% of the students stated that the approach concretized the subject and 25% of them mentioned that the approach made them like the subject. S5 and S9 shared their views on the topic:

"I think that it has benefits for concretizing abstract concepts."

"Coming to the class prepared helps me to eliminate the prejudices against the course. Thus, I liked integrals. Integrals Makes me happy."

When students' views were examined, it was revealed that the concepts before the course became more concrete with this approach and this caused students to learn the subject more easily. The views of S13 and S19 supporting this opinion can be given as examples:

"We have to come to the lesson prepared with this model. This enables us to understand much more easily and quickly." (S3)

"Usually students have difficulty with forming concepts. Because flipped mathematics classroom and software turn the abstract concepts into concrete ones, it is much easier to understand them..." (S19)

Only 28% of the students stated that it was possible to learn difficult mathematics concepts simply with the flipped approach designed using Khan Academy materials and mathematics software. The views of S16 can be given as an example:

"It teaches how to learn as it presents the complex subjects in a simple way. As I told, complexity is reduced to simplicity..."

Only 21% of the students stated that the approach moved them away from memorization. S3 shared her opinion that:

"I learned the double integral rationally because of the flipped mathematics classroom. It is so clear that it is against rote learning and lecturing."

Students stated that the visualization provided with this approach promoted meaningful learning rather than rote learning. A student's view can be given as an example:

“Khan Academy visualises learning and concretizes abstract concepts... The abstract and difficult concepts to conceptualize in our mind can be seen easily and understood better with mathematics software.... Thus, flipped model is more effective in meaningful learning due to understanding and visualizing than rote learning.” (S26)

When Table 2 was examined regarding “suggestions,” most of the students stated that in the flipped classroom approach, the Khan Academy materials and mathematics software should be used together. Moreover, the students stated that they wanted this approach to be used in all subjects. The following views of two students belonging to this category can be given as examples.

“Khan Academy teaches learning because it teaches the complex subjects in a simple way... The model and Khan Academy are ideal to teach difficult subjects easily. As I said, complexity is reduced to simplicity. There is only one thing missing here. The lack of visuals (graphs, diagram, etc.) are completed by GeoGebra and Maxima. All three (GeoGebra, Maxima and Khan Academy) go well together.”(S4)

“It is a very good model. In my opinion, it must be used with all subjects.”(S2)

When the “disadvantages” were examined in Table 2, five students stated that they had difficulty due to lack of information about how to use a computer while two students stated that they had difficulty with using mathematics software. Two students’ views can be given as examples of this category:

“I had difficulty because of lack of information about how to use a computer.” (S22)

“I had difficulty with generating graphics with software.” (S26)

Discussion

Within the context of this study, students successfully used the Khan Academy materials related to the use of double integrals outside the classroom. They learned about the double integral in class through informal cooperative activities using materials and worksheets which the researcher had designed and developed from the literature using GeoGebra and Maxima. Quantitative and qualitative data were collected during the implementation of the approach in the research. The analysis of qualitative data demonstrated that students understood the concepts much better as a result of using the flipped classroom approach that was designed using the Khan Academy materials and mathematics software together. In addition, it was emphasized by the students that the approach helped them to visualize the concepts, promoted retention of knowledge, and fostered easier learning of the concepts. Moreover, it was revealed that the approach helped students to come to the lesson prepared, it concretized the subject and made students like the subject. These findings are similar to the results of the study carried out by Sahin, Cavlazoglu and Zeytuncu (2015).

Another finding from the qualitative data was that the approach might move students away from memorization. Considering the views of S3, S25, and S28, it was determined that coming to the class ready with this approach enhanced students’ understanding of the subject better and much easier and increased their motivation. Moreover, it was determined that with this approach, using worksheets in class with the Khan Academy materials and the mathematics software together made lessons more visual and concrete for students. From this point of view, it can be stated that students understood mathematics much better and more easily since the well-prepared students discussed the concepts more concretely and visually. It was found based on the views of S9 and S11 that the visuals obtained with this approach promoted retention of knowledge. Teaching concepts in a more visual environment might move students away from memorizing and learning can be actualized in a more conceptual learning environment. Especially the view stated by S26 who emphasized that by using this approach, mathematics was learned in a meaningful way rather than by memorizing, supports this opinion. When the qualitative data obtained were examined, it was found that well-prepared students studied mathematics in a more concrete and visual environment and thus, understood concepts much better and more easily. The students’ views revealed that visualization offered opportunities for students to learn the concepts more permanently and also to move away from memorizing; it provided a meaningful learning environment. It can be stated that these positive contributions gained with this learning approach increased students’ achievement in the learning of the double integral. The quantitative data obtained revealed that the flipped classroom approach designed using the Khan Academy materials and mathematics software together promoted student achievement. The results obtained in this research correspond to other studies that have been conducted on the flipped classroom approach in mathematics teaching (Fulton, 2012; Love, Hodge, Grandgenett, & Swift; 2014; Sahin, Cavlazoglu, & Zeytuncu, 2015; Van Sickle, 2015). The evaluation carried out based on both quantitative and qualitative data obtained in the research study are given in Figure 4.

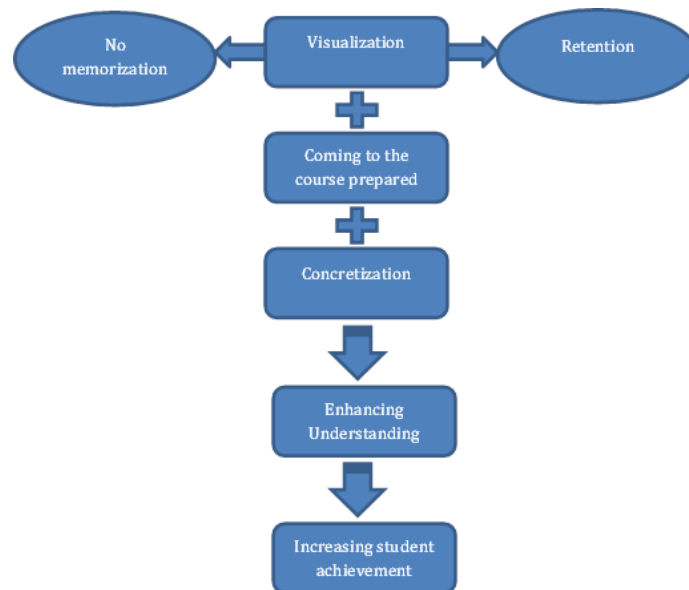


Figure 4. Evaluation of the flipped classroom approach based on qualitative and quantitative data

The flipped classroom approach designed using the Khan Academy materials and mathematics software provided students with the opportunity to study mathematics in a visual and concrete learning environment. Students who came to the class were prepared to take the opportunity to study mathematics in a concrete environment using static concepts in a dynamic and visual environment with the support of the Khan Academy materials and mathematics software. Thus, the difficult mathematics topics are taught in a simpler way. S16 emphasized that she learned about the double integral using this method while she was previously having great difficulty in understanding the definite integral concept. The flipped classroom approach supported with the Khan Academy materials and mathematics software enabled students to understand mathematical concepts much better and more easily and thus students' achievement increased. Although this approach made positive contributions, some students had difficulties due to their failure to use a computer and software. Therefore, it is considered that the implementation of this approach in the classroom setting becomes a disadvantage for students who do not know how to use a computer.

Conclusion and suggestions

The study examined the effect of the flipped classroom approach designed using the Khan Academy materials and mathematics software on student achievement of the topic of double integrals and elicited students' views about this approach. With regard to RQ1 (What is the effect of the flipped classroom approach designed by using the Khan Academy materials and mathematics software on student achievement in the double integral topic?), the qualitative and quantitative data obtained demonstrate that this approach is an effective model in increasing student achievement. In addition, when not only the differences between pre-test and post-test scores but also the mean ranks were compared, it can be stated that this approach increased student achievement. Moreover, based on students' views with respect to RQ2 (What are students' views on the use of the Khan Academy materials and mathematics software with the flipped classroom approach in mathematics courses?), it was found that this approach enhanced students' understanding of the mathematical concepts, it made the course more visual, and it promoted retention. Besides the three main contributions of this approach, it was revealed that this approach made understanding much easier. Therefore, the contributions of the approach may promote the achievement of students.

In this research, the flipped classroom approach was supported with Khan Academy materials outside of the classroom and with free open source software like GeoGebra and Maxima that were used in the class. It was found that this approach supported with using the Khan Academy materials and free open source software together can be an effective model for a student anywhere in the world to increase his/her achievement in a course with a high level of difficulty like mathematics. Teachers who want to implement the flipped classroom approach in their lessons can benefit from the Khan Academy materials instead of preparing the contents themselves. There is a lot of free open source software offered for use in class activities. GeoGebra and Maxima used within the context of this study offered a dynamic learning environment to students thus providing opportunities for visualization and concretization. The teachers who are going to use the flipped classroom

approach can enrich their instruction with such software. So, if the flipped classroom approach is not supported with such software programs and worksheets, the strengths of the approach and its contribution to students' understanding may not emerge. It can be much easier for students who study the course outside of the class with platforms like the Khan Academy to enrich their conceptual knowledge using mathematics software in class. Therefore, it is suggested that the flipped classroom approach should be used supported with such software programs.

Limitations

The research has two fundamental limitations. One of them is related to the quantitative side of the research. The quantitative part of the study was designed with a single-group. Because there was only one group, comparison was made only within the group. If a randomized pretest-posttest control group design or a non-equivalent pretest-posttest control group had been used, the data of the experimental group would have been compared with the control group data. Another limitation of the study is that the study group was determined by the convenience sample method. As the researcher does not have the opportunity to work with a large number and diversity of participants, this limitation has emerged. Furthermore, the self-reported data presented here may cause students to focus only on limited aspects of the approach. As seen in the results, students mostly focused on positive effects of the approach rather than negative effects. While this enriched the results of the study by highlighting the advantages of the study, it also limited identifying disadvantages of the approach.

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Appendix 1

An Example of Worksheets Contents

You can ask for advice from teacher about the content and the software. Carry out your work with your pair.

- Using GeoGebra, explain Fubini's Theorem. Use input, graphics, and GeoGebra tools. In addition, you can use 3D graphics.
- Open material M2. Interpret how to obtain the volume in double integrals with the help of material M2.
- State the integration region of the following integral with the help of GeoGebra where R is bounded triangle with x -axis, $y=x$ and $x=2$ lines

$$\iint_R f(x, y) dA$$

- On the region R , find integration bounds for $\iint_R dA$;
 - by using vertical lines
 - by using horizontal lines

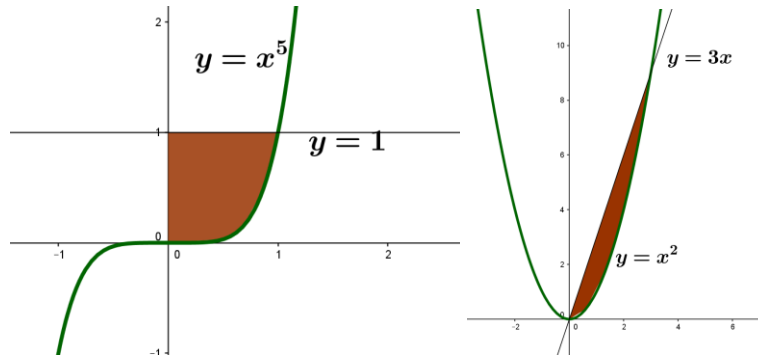


Figure 5. A screenshot belonging to the worksheet

- Find the volume of the region which is bounded above by the paraboloid $z = x^2 + y^2$ and bounded below by the triangle formed from the lines $y=x$, $x=0$ and, $x+y=1$. Verify the obtained result with maxima. Draw the surface with maxima
- For the following integral

$$\int_0^{3/2} \int_0^{9-4x^2} 8x dy dx$$

- sketch integration region
- write an integral which is equivalent to integral whose integration order is changed
- find the value of integral with maxima
- Define a surface with the help of maxima and find the volume below the surface in the integration bounds.