Middle School Students' Mathematics Knowledge Retention: Online or Face-to-Face Environments

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ABSTRACT

Educators seek to develop students' mathematical knowledge retention to increase student efficacy in follow-on classwork, improvement of test scores, attainment of standards, and preparation for careers. Interactive visuals, feedback during problem solving, and incorporation of higher-order thinking skills are known to increase retention, but a comparison of online and face-to-face learning regarding knowledge retention has not been fully explored. The current study tested 38 Caucasian eighth grade students for knowledge retention on ten mathematical topics they had learned in sixth grade during either online or face-to-face conditions. After two years, students were given the same posttest as in 6^{th} grade over the 10 units of which five had been learned online and five face-to-face. Scores for long-term gain scores showed no significant differences between online or face-to-face learning conditions. A TOST calculation was used to show equivalence of middle school student long-term learning across online and face-to-face conditions in mathematics in this study within the limits of the test. The interactive nature of each condition contributed to strong mathematical understanding, which led to the retention being equivalent for both conditions.

Keywords

Mathematics, Knowledge retention, Middle school, Math, Comparison of online and face-to-face learning

Introduction

Retention of knowledge is vital for future application of concepts. Because knowledge deteriorates after long periods of non-use, it is important to determine the best ways to instruct students so that they retain information and skills (Hall, Stiles, & Horwitz, 1998). Several studies on knowledge retention have been conducted in the fields of medicine and military science in particular. In the medical profession, a patient's life may be in the hands of a surgeon who has to make a quick decision when an unexpected complication occurs. A medical study focusing on retention of knowledge of diabetes showed that knowledge loss can happen in as little as a week if the new learning is not reinforced often (Bell, Harless, Higa, Bjork, Bjork, Bazargan, & Mangione, 2008). Another medical study (Naidr, Adla, Janda, Feberová, Kasal & Hladíková, 2004) showed that one year later, students retained only 66.8% of their previously tested learning.

In the military field, a country's safety may hinge on a critical choice made by a soldier or group under the pressure of combat. This situation was examined in a military retention study (Ricci, Salas, & Cannon-Bowers, 1996). In this investigation, even a 25 day-lapse after learning concepts related to aviation antisubmarine warfare or basic electricity and electronics resulted in a 24% loss in knowledge of these areas. While K-12 education topics do not tend to address immediate life or death matters, knowledge retention is important for students to score well and gain college admittance to pursue their desired careers as well as being crucial to the survival of school districts in this era of high stakes testing of student knowledge.

Knowledge retention in mathematics education

Researchers in mathematics knowledge retention have attempted to identify key learning components that promote long-term learning. One investigation compared student diagram construction versus logical argument in learning geometry proofs (Lovett & Anderson, 1994), finding that work with diagrams rather than logical explanations of the problems resulted in stronger memory for rules of geometry proofs. Another study (Butcher & Aleven, 2008) examined long-term learning in which tenth grade students used a computer application that provided hints and feedback as students solved geometry problems compared to student interaction with labeled geometry diagrams. These researchers found that visual information needed to be integrated into problem-solving practice to support knowledge retention. A third investigation examined the effects of reform mathematics teaching versus more traditional mathematics instruction (Edwards & Townsend, 2012), concluding that work involving higher-level thinking and student interaction in the problem solving process produced higher standardized test scores, which are valued data in today's educational landscape. A comparison that has not been researched as thoroughly is long-term learning in an online environment versus a face-to-face setting, especially for middle school students. Because of this need for empirical results related to online learning and mathematical knowledge retention coupled with the increasing amounts of online learning occurring in today's educational system (Aharony & Bronstein, 2014; Bejerano, 2008; Young, Birtolo & McElman, 2009), the researchers conducted a study to test equivalence of the conditions. As Greener (2013) asked, is digital learning "supporting learning better than its physical precursor?" (p. 417).

Literature review

Online and face-to-face retention comparisons

Few studies have been conducted to show knowledge retention comparisons between online and face-to-face conditions, especially in the area of mathematics. The following review of the literature highlights studies comparing an online condition to a face-to-face condition regarding knowledge retention.

One study that compared knowledge retention under online versus face-to-face learning involved university medical students in an anatomy and physiology class (Rondon, Sassi, & de Andrade, 2013). In this study, students were randomly selected to attend either a course utilizing computer games as instruction (online learning), or a course involving traditional classroom (face-to-face) lecture and discussion. To gather retention data, the researchers gave a pretest before the course, an identical test at the conclusion of the nine-week course (posttest), and an identical test six-months after the conclusion of the course (long-term posttest). Regarding short-term gains from pretest to posttest, both conditions showed no significant difference in gain scores. However, the face-to-face group's gain scores from pretest to long-term posttest revealed a significantly *smaller* loss of previous learning in the long-run.

Researchers of another retention study (Ricci et al., 1996) tested 60 students in a naval training course learning about chemical warfare. The students were split into three groups. One group read a set of printed lecture notes, another group studied a book, and the final group used an online computer program. Identical pretest, posttest, and retention tests (administered four weeks after the posttest) were given. Students in each condition improved in knowledge from the pretest to the posttest in a statistically significant manner ($p < .001$); however, only the online group showed a statistically significant positive difference in retention from posttest to retention test ($p < .05$). Therefore, this study, in contrast to the previously-discussed anatomy and physiology class experiment, retention favored the online condition.

In a K-12 education example, a study was conducted with a Turkish high school biology class to compare the use of online hypermedia materials with traditional lecture (Yildirim, Ozden, & Aksu, 2001). Both conditions allowed students to engage for 15 hours with the information. The students were given a pretest to set a baseline. A posttest was given immediately at the conclusion of the biology course, and the retention measurement component was administered one month after the conclusion of the class. On the long-term retention test, the control lecture group lost points compared to their previous posttest score, while the hypermedia class gained points. The difference in retention of both conditions was found to be statistically significant ($p < .05$), favoring the online condition.

A final retention study example involved a medical-surgical nursing class separated into two conditions: students instructed through a computer-assisted learning module versus those attending a more conventional lecture and demonstration (Fernandez Aleman, Carrillo de Gea, & Rodriguez Mondejar, 2011). Investigators administered a baseline pretest to determine initial knowledge. After a ten-week lapse, positive retention occurred in both conditions with no statistically significant differences in student scores.

Overall, in the current literature, there are studies that favor online or face-to-face learning or neither condition. To provide more empirical information about long-term knowledge retention regarding online mathematical learning, the authors conducted the current study. This investigation examined knowledge retention of eighth grade mathematics students who had previously experienced both online and face-to-face instruction in learning ten mathematics topics during the sixth grade.

Interactive learning and knowledge retention

Earlier theories of knowledge acquisition (Anderson, 1982; Fitts, 1964; Rasmussen, 1986; VanLehn, 1996) can be consolidated into a three-phase process model of learning (Kim, Ritter & Koubek, 2013). This model has these stages: (1) obtain declarative and procedural knowledge, (2) combine and solidify the acquired knowledge and (3) fine-tune the knowledge through overlearning. During the first stage, students learn facts and concepts that support ways of using those facts; in the second stage, this knowledge is consolidated into a procedure for applying the information; and finally, during the third stage, practice allows students to speed application of the procedures (Kim et al., 2013). The fine-tuning stage not only produces greater fluency of knowledge recall, but the number of distinct days that a student practices a particular mathematical skill is important to long-term knowledge retention (Wang & Beck, 2012). This result confirms a finding in the field of vocabulary (Pavlik & Anderson, 2005), that wide spacing of practice provides increasing benefit as practice accumulates resulting in less loss of knowledge later. Similarly, the author of a review of long-term retention of science knowledge (Custers, 2010) suggested the following strategies gleaned from the reviewed literature, to increase long-term retention of learning: (1) brief appropriately-spaced practice sessions; (2) avoidance of prolonged intensive study, instead interspersing of short bouts of intensive study; (3) periodic review of concepts already studied or mastered; and (4) regular quizzing and testing.

More-interactive methods of instruction have a positive effect on student mathematical knowledge retention. A study that involved college students who were learning Cantor set theory (Narli, 2011) found that students taught with an interactive, Constructivist methodology compared to traditional lecture, retained more mathematics information 14 months later than the control group. A study of 60 ninth grade students in Turkey (Guvercin, Cilavdaroglu, & Savas, 2014) showed that students who engaged in mathematical problem posing had better achievement, attitudes, and retention of knowledge 40 days after conclusion of instruction. Finally, a meta-analysis of 55 studies that had participants from kindergarten through college compared the use of concrete manipulatives to the control condition of discussion using abstract math symbols of concepts (Carbonneau, Marley, & Selig, 2013). These studies showed that overall there was a small to medium positive effect size of using manipulatives; a separate analysis of knowledge retention using manipulatives showed a large effect size.

Computer games can provide an active, multisensory learning environment; therefore their potential to increase student engagement and learning is great. The individualized mass and distributed practice of computerized games supports the different stages of learning in Kim, Ritter and Koubek's theory (2013) previously described. A study of games use, compared to traditional lecture, in teaching anatomy and physiology of the head and neck to college students (Rondon et al., 2013) showed overall no significant difference between conditions for the short term, but an advantage for face-to-face instruction in the long term. Those investigators surmised that the ability of students using texts in the face-to-face condition to go back and review information as desired influenced the results and suggested that games be altered to provide this ability. In contrast, a study of fourth graders' mathematical learning of multiplication and division of natural numbers and fractions (Pilli & Aksu, 2013) showed that the experimental group using the computer software performed better than the traditionally taught control group on all areas of the posttest, and performed better in the long term on operations involving natural numbers, but not fractions. Other experimental–control group research addressing seventh grade students' misconceptions about probability (Gürbüz & Birgin, 2012) indicated that computer-assisted teaching was significantly more effective than traditional methods on a posttest, but the researchers did not provide long-term learning effects. This attention to misconceptions can assist students in solidifying and fine-tuning their knowledge.

The current study investigates long-term knowledge retention of middle school students who participated two years earlier during sixth grade in a counterbalanced, interactive online and interactive face-to-face mathematics study in which ten mathematics topics which were part of the district's curricular plan (prior to full implementation of the Iowa Core) were taught with students experiencing five topics under each condition. The online condition incorporated virtual manipulatives, online lecture videos, online simulations and interactive diagrams, online games and peer interactions through an online chat. Similarly, the face-to-face condition incorporated hands-on manipulatives, live lectures and explanations, visuals and diagrams, games, and group work with discussion. Both conditions were designed and taught by the same instructor. Posttest results at the end of sixth grade indicated that learning results of the two conditions were equivalent (Edwards, Rule, & Boody, 2013).

Methods

The current study examined the retention of mathematical knowledge two years after a series of posttests given in a prior study (Edwards et al., 2013) comparing the long-term knowledge retention of sixth grade (now eighth grade) students in face-to-face and online conditions in a mathematics classroom.

Participants

The participants in this study included 38 Caucasian eighth graders, 20 male, and 18 female from the same rural Iowa school district. The original sixth grade study had 46 Caucasian students; an attrition of five males and three females from the original to the current study. No additional students were added to the study. This study was approved by the committee for research on human subjects at the researcher's university. Fully-informed, written school district, parent, and student consents to participate in the study were obtained.

Study design

The eighth graders had previously been divided into mixed-ability groups in sixth grade by the district's fifth grade teachers with no knowledge of an eventual study. While the students (as eighth graders) were no longer grouped this way for courses, the data were analyzed using the previous groupings from sixth grade. As eighth graders, 16 of the students had their former sixth grade mathematics teacher as the primary mathematics instructor (7 male/9 female) while the other 22 students had an alternate instructor for either regular eighth grade math or Algebra I (13 male/9 female). Because this research followed a quasi-experimental design, conclusions were established on the variables implemented (Clark & Creswell, 2010); in this study, the independent variable was the type of instruction: online or traditional face-to-face teaching during sixth grade. The dependent variables were student academic retention shown by long gain scores (the change from the original sixth-grade posttest taken two years ago to the final eighth-grade posttest score) and the final eighth-grade posttest scores. The sixth grade mathematics topics were not specifically retaught during seventh and eighth grades, as the state-required standards addressed new concepts rather than revisiting the old ones.

Posttesting occurred during the last month of the students' eighth grade year. Some sample questions taken from the probability unit include P(multiple of 3) given a set of cards numbers 1-10 and P(7) given a standard die (Foresman & Wesley, 1998, p. 628). The ten final eighth-grade posttests were the exact same tests as the students answered throughout their sixth grade year as part of the previous study (Edwards et al., 2013), and covered the topics of decimals, statistics, algebra, probability, measurement, symmetry, geometry, polygons, perimeter, and area (all topics listed in the order they were taught and decided upon by the district's curricular focus at the time the students were in 6^{th} grade). During the previous study, the students participated in ten units, alternating between an online condition and a face-to-face condition in a counterbalanced design. The online condition had students sit separately at desks in the classroom with individual laptop computers during mathematics class time. Learning modules were set up online that included videos and/or electronic manipulatives and interactive sites to explain concepts. Students were not allowed to communicate face to face; instead, they used a chat program to interact with the instructor and other students. Instant messaging has been shown to be an effective way to support informal mathematics coaching (Hrastini, Edman, Andersson, Kawnine, & Soames, 2014). Students completed the same book assignments and used feedback videos to check and see how well they understood the material. Students turned these assignments in electronically, digitally asked any questions they might have had, and were free to move on to the next section in the module.

Students in the face-to-face condition also sat individually at desks in the classroom. The instructor explained the concepts and utilized manipulatives and interactive activities to reinforce understanding. The teacher assigned the same book assignments as used for the online condition. Students could collaborate with other students or ask the instructor questions about the concepts verbally. Students received feedback on completed assignments the following day. Students who completed the day's work had other enrichment or problem solving opportunities but could not move on to the next section.

The same instructor in the same classroom oversaw both the online and face-to-face environments. Two different sections of students were involved in the original study. One class learned a mathematical topic face-to-face for a few weeks, while the other class learned the same topic in an online manner. Classes switched conditions for each topic, thereby learning five of the ten topics under each condition. No homework was given for either of the conditions, allowing the focus to be on working during each class period. When online manipulatives or interactive games were used, a similar version was presented in both conditions. For example, the online condition utilized the National Library of Virtual Manipulatives (Utah State University, 1999) to show equal fractions through the use of virtual pie pieces while the face-to-face students had physical colored pie slices to put together manually to make the comparison.

The instructional focus of both conditions was mathematics for understanding. A premium was placed on students being able to explain the underpinnings of the material being presented in both formats. To ensure the effectiveness of this approach, both conditions regularly utilized manipulatives, interactive activities, peer discussion, teacher explanations, and the requirement of students to demonstrate understanding for completed work. While the instruction occurred on video for the online condition and in person for the face-to-face environment, both situations had the same experienced instructor who explained the meaning behind the mathematics rather than memorizable procedures. This interactive pedagogical approach is necessary for deep learning to occur (Narli, 2011). Information pertaining to materials and methods used for both conditions from a portion of the probability unit can be found in Table 1.

Table 1. Activities for both the face-to-face and online conditions in the probability unit

Activity	Face-to-face condition	Reference	Online condition	Reference
Instruction	Standard Direct Instruction (Similar Content to the Video for the Online Condition) by the Teacher with Discussion	NA	Online Instructional Video Created by the Teacher	Edwards, 2010a
Manipulatives	Physical Coins and Decks of Cards	NA	Coin and Card Simulations	Random.org, 1998a; Random.org, 1998b
Book Problems	Textbook Problems	Foresman & Wesley, 1998	Textbook Problems	Foresman & Wesley, 1998
Checking the Problems	Teacher Walks Around Checking/Asking Questions to Students/Groups/Adding Insight When Needed	NA	Online Instructional Video Created by the Teacher for Students to Check Work and Gain Additional Understanding	Edwards, 2010b
Communication	Teacher to Student and Student to Student Conversations	NA	Teacher to Student and Student to Student Google Talk	Google, 2011

Original posttests were taken at the end of each trimester to compare the conditions (online and face-to-face) to see if one was better for a student's learning than the other. The current study is an extension of the previous study, focusing on long-term retention of mathematical content and procedures learned under each condition.

Instrumentation

The ten posttests each contained ten questions and were adapted from the *Middle School Math Course One* sixth grade textbook used in the school district (Foresman & Wesley, 1998). This text followed the state and local standards. The posttests each contained ten questions of varying difficulty levels. Students needed to demonstrate mathematical understanding for a response to be scored as correct. Tests were untimed, on paper, and scored by the teacher, as in the original study to ensure scoring continuity.

Data analysis

Student long gain scores and new, final eighth-grade posttest scores were analyzed topic by topic. The scores of the class experiencing the traditional face-to-face method were compared to scores of the class learning through the online methods for the same topic. A *t*-test was applied to scores earned under both learning conditions for each of the mathematics topics to determine if there were significant differences in learning between long gain scores and new posttest scores.

In the previous study, students participated in five of the ten topics for each learning situation (online or face-to-face). The five scores from each condition were totaled to generate a composite score for each student. Each composite score had a maximum of 50 possible points (a 10 on each of the five sections of the pretest-posttest results in a total score of 50). Using this information, the equivalence of retention for online learning of mathematics and retention of face-to-face learning of mathematics was tested.

Results and discussion

Long gain scores by mathematics topic

The long gain scores (the change from the original sixth-grade posttest taken two years ago to the final eighth-grade posttest score) for all ten units of instruction are shown in Table 2. These long gain scores are fairly small because during sixth grade, most students had mastered much of the mathematics material presented and did not have as much room for growth as they originally did. Table 2 shows three places at which students evidenced negative gain scores (a loss of previously-learned content). Students learning perimeter online and the topics of area and polygons under the face-to-face condition forgot some of the mathematical concepts they had previously successfully applied during the sixth-grade posttests. The perimeter concept students had the most difficulty remembering concerned the idea that a circle’s perimeter is approximately three times the diameter. Students who originally studied area in the face-to-face condition seem to have forgotten some of the material; they especially scored low on complex area problems of calculating area of figures with straight and circular sides. The students who originally studied these concepts in the online condition may have had an advantage in working with more interactive online diagrams that allowed students to disassemble the complex figure and analyze each part.

There were no statistically significant differences between long gain scores of eighth grade students who originally studied these topics during the online and face-to-face conditions of the original sixth-grade study. This indicates that, in general, students taught in an online condition or in a face-to-face condition seemed to have built a similarly strong foundation of knowledge in these topics and to have retained similar amounts of mathematical information.

Table 2. Long gain scores for each separate topic

Order	Mathematics topic	Trimester taught	Class group	*n*	Mean long gain score		*SD*	*t*	*p*
					Online	Face to face			
1st	Decimals	1	A	19		1.11	1.63	.48	.63
			B	19	.79		2.35		
2nd	Statistics	1	A	19	2.18		3.18	.71	.48
			B	19		1.55	2.19		
3rd	Probability	1	A	19	.53		2.22	-1.44	.16
			B	19		1.50	1.94		
4th	Algebra	1	A	19		1.95	1.55	.25	.80
			B	19	1.76		2.80		
5th	Measurement	3	A	19	.47		1.70	-.93	.36
			B	19		1.00	1.83		
6th	Symmetry	3	A	19		.21	1.40	.00	1.00
			B	19	.21		1.84		
7th	Geometry	3	A	19		.74	1.33	-.74	.47
			B	19	1.16		2.12		
8th	Polygons	3	A	19	.11		2.00	1.33	.19
			B	19		-.79	2.15		
9th	Perimeter	3	A	19	-.53		1.61	-1.85	.07
			B	19		.47	1.71		
10th	Area	3	A	19		-1.47	2.93	2.02	.051
			B	19	.26		2.33		

Final eighth-grade posttest scores by mathematics topic

Table 3 lists the new final eighth-grade test scores for each of the ten different topics. The only unit of instruction that showed a statistically significant difference between the two conditions regarding final eighth-grade scores was the unit on probability. Class A (originally in the online condition for this topic in sixth grade) scored quite a bit lower than Class B (a difference of 1.31 points on the 10-point test). Both classes scored very similarly on probability on the original sixth-grade posttest (Class A's score was 5.79; Class B's score was 6.13).

Table 3. New final eighth-grade test scores for each separate topic

Order	Mathematics topic	Trimester taught	Class group	*n*	Mean long gain score Online	Face to face	*SD*	*t*	*p*
1st	Decimals	1	A	19		7.63	1.67	-1.12	.27
			B	19	8.21		1.51		
2nd	Statistics	1	A	19	6.53		2.37	-.30	.76
			B	19		6.74	1.88		
3rd	Probability	1	A	19	6.32		1.86	-2.18	.04
			B	19		7.63	1.86		
4th	Algebra	1	A	19		8.05	2.04	-.60	.55
			B	19	8.42		1.71		
5th	Measurement	3	A	19	6.89		1.63	-.82	.42
			B	19		7.37	1.92		
6th	Symmetry	3	A	19		8.11	2.08	.08	.94
			B	19	8.16		1.95		
7th	Geometry	3	A	19		6.74	2.10	.07	.95
			B	19	6.68		2.52		
8th	Polygons	3	A	19	7.37		1.61	1.64	.11
			B	19		6.26	2.47		
9th	Perimeter	3	A	19	5.84		2.04	-.42	.68
			B	19		6.11	1.79		
10th	Area	3	A	19		3.63	1.83	-1.31	.20
			B	19	4.42		1.90		

Comparing the total of all online to the total of all face-to-face scores by student

In this part, long gain score totals will be discussed first, followed by final eighth-grade posttest score totals. The reader will recall that the posttest consisted of a set of ten tests, one on each mathematics topic. In the original study, students learned five topics online and five topics face-to-face. Because the study was counterbalanced, topics learned online by one class were learned face-to-face by the other class. A paired-samples *t*-test was conducted comparing each student's total of the long gain scores for the appropriate five mathematical topics for the online condition with the total of the long gain scores for the appropriate five mathematical topics learned in the face-to-face control condition. It was nonsignificant (Mean of the paired differences = .11, $t(37) = .10$; $p > .05$, Standard Error of the Mean = 1.05).

Recognizing that a lack of significance does not necessarily imply equivalence but may simply indicate insufficient power (Hoenig & Heisey, 2001), the researchers went further to directly test equivalence concerning the long gain score totals. Following the TOST (two one-sided *t*-tests) approach (see, for example, Chen, Rathore, Ji, & Germansderfer, 2010), we first developed a "zone of indifference": the width of scores within which scores could be taken as equivalent. This is not itself a statistical matter; it should be developed directly out of the context. The teacher set the zone, based upon years of experience assessing students' performances connected to their understandings of mathematics. Based on his report, we chose ± 2.5 points (out of the 50 points each students could have earned across 5 topics). Thus, for the score 40, 42 would be considered equivalent, but 43 would not be considered equivalent to 40.

The zone of indifference for the long gain scores, then, would be 1.5 pts. ± 2.5 pts., or -1.0 to 4.0. We then implemented the two one-sided *t*-tests approach by developing a 90% confidence band around the mean of the difference scores. For the gain scores, this band was $.54 \leq 1.50 \leq 2.46$ (the mean of the differences (1.50) ± .96 (which is the standard error of the mean (.78) x $_{.90}$CI $t(37)$ (1.687)). This suggests that the long gain scores for online and face-to-face were not equal (statistically different) but can be considered equivalent within the level of indifference chosen.

For the new, eighth-grade posttest score totals, a paired-samples *t*-test between online (total of the appropriate 5 new test unit scores) and traditional face-to-face (total of the appropriate 5 new test unit scores) was calculated. It was significant (Mean of the paired differences = 1.50, $t(37) = 2.61$, $p = .013$, Standard Error of the Mean

= .57). In this sample, at least, students who were originally taught online in sixth grade, compared to students originally learning face-to-face in sixth grade, had higher final eighth-grade test scores in the current study two years later. Although the results of this test indicated a significant difference in eighth-grade posttest scores between students who had originally learned these topics online or face-to-face during sixth grade, this difference was small and may possibly fit within the zone of indifference for the TOST. Therefore a TOST was conducted on eighth grade posttest score totals. This statistical difference may be influenced by loss of subjects: the original sixth-grade study involved 46 students, but by eighth grade, 8 of these students had moved to other schools, eliminating them from the eighth-grade posttesting.

For the TOST on the new eighth-grade posttest scores, the zone of indifference would be .11 ± 2.5 pts., or -2.39 to 2.61. The band provided by the TOST analysis was $-1.66 \leq 0.11 \leq 1.88$ (the mean of the differences (.11) ±1.77 (which is the standard error of the mean (1.05) x .90CI $t(37)$ (1.687)). Because the entire confidence interval in each of these two cases (for gain scores and for posttest scores) is contained within the zone of indifference, and there was no statistical difference from this TOST, the researchers conclude that the achievement as measured by the new eighth-grade posttest scores from the online and the traditional learning conditions may be considered as equivalent at the .05 level of significance.

Conclusion

Summary

This study was conducted to determine whether retention of mathematics knowledge in an online mathematics environment was equivalent to retention of mathematics knowledge in a face-to-face mathematics environment for middle school students as shown through posttest performance two years later. Individual *t*-tests used on each topic comparing the online condition with the face-to-face condition in regards to long-term gains (retention from sixth grade to eighth grade) showed no significant difference for any topic.

Next, a *t*-test comparing student long-term gain (retention) and a *t*-test comparing new posttest scores under both conditions (total scores for five mathematical topics) were used showing no significant difference in long-term gain, but a significant difference in final posttest scores in favor of online learning. Two additional TOST analyses were conducted to evaluate equivalency for both long-term gain scores and for final posttest scores. The results of the TOST indicated that the difference occurred between the confidence intervals, meaning that both the online and face-to-face conditions may be considered equivalent for both long-term gain scores and final posttest scores within the established zone of indifference. This information shows that online learning can be a viable method of learning for middle school students in regards to both mathematical knowledge retention and achievement.

Interactive learning

The instruction sixth grade middle school students experienced during the face-to-face and online conditions had many interactive components including virtual or hands-on manipulatives; online simulations with interactive diagrams or visuals and diagrams with live explanations and discussion; online games or hands-on games; and peer interactions via a chat center or face-to-face group work. These interactions allowed students to satisfy the three phases of Kim et al.'s (2013) model of learning. Students learned facts and concepts from online or live lectures and textbook readings, then, they combined and solidified that knowledge through interactive manipulatives, simulations, diagrams, and problems solved with peers via group work in chats or face-to-face, and finally, they fine-tuned their knowledge through additional practice during online or face-to-face games. Both the online and face-to-face conditions supported student learning through interactivity contributing to equivalent knowledge retention across the conditions and supported the findings of Carbonneau, Marley, and Selig (2013), Guvercin, Cilavdaroglu, and Savas (2014), and Narli (2011).

Recommendations for future research

While this study showed mathematical retention in middle school students to be equivalent in both online and face-to-face conditions, more questions remain. Similar studies expanded to different student age groups might indicate optimal ages for online learning. The current study showed middle school students evidenced equivalent mathematical retention in both online and face-to-face situations; would younger populations produce similar results? Is there a point when students are too young to enjoy the benefits that mathematical online learning can bring? Students in the original sixth grade study (Edwards et al., 2013), who had experienced both conditions, reported enjoying online learning more than face-to-face instruction as long as the online learning was interspersed with face-to-face interactions. Do students feel the same way in the long term? Does a more prolonged exposure to online learning throughout middle school and high school cause that enjoyment to wane?

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